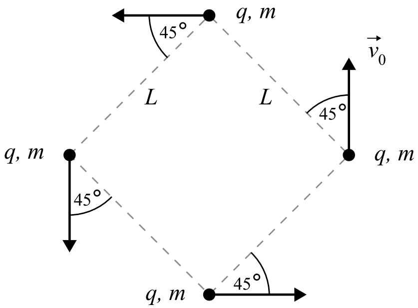
NORDIC-BALTIC PHYSICS OLYMPIAD 2024

1. FOUR CHARGES (7 points) — *Päivo Simson, Jaan Kalda.*

Four identical particles are initially in the corners of a square, as shown in the figure below. All particles have the same charge q, mass m, and the same magnitude of initial velocity v_0. The directions of the initial velocities are indicated in the figure. You can assume $v << c$ and ignore gravity.



i) *(2 points)* After a long time has passed, what is the magnitude of the final velocity v_f of the particles with respect to the center of mass of the system?

ii) *(5 points)* What is the angle between the initial velocity $\vec{v}_0$ and the final velocity $\vec{v}_f$ of a particle?

2. OKLO FISSION REACTOR (7 points) — *Topi Löytäinen, Jaan Kalda.*

Based on the ratio of the uranium isotopes ^{235}U and ^{238}U, as well as the abundances of the isotopes produced by nuclear reactors, researchers have established that self-sustaining natural nuclear reactors operated in Oklo ca $T_0 = 1.8 \times 10^9$ years ago in Gabon, central Africa. For such reactors to exist, two conditions must be met: (a) presence of deposits with high enough concentration of uranium; (b) sufficiently high abundance of ^{235}U in natural uranium. Rich uranium ores were created by floods: scattered uranium was dissolved in oxygen-rich water and transported by it to underground pools. Significant concentration of oxygen appeared in the atmosphere only around 2.5 billion years ago, so the first condition was not met earlier than that. You will learn below that the abundance of ^{235}U decreases relatively fast in time, so the second condition ceased to be satisfied soon after the operation of Oklo's reactor.

What made the operation of Oklo's reactor possible was a stable influx of ground water that kept the uranium deposits sufficiently wet. Water is the so-called moderator for the fission reactor: it slows down neutrons emerging from fission reactions, dramatically enhancing the chances of a neutron triggering the fission of a next ^{235}U nucleus.

In what follows, in addition to T_0, you can use the following numerical values.
Energy released by the fission of a single ^{235}U nucleus: $E_0 = 200\,\mathrm{MeV}$.
Half-life of ^{235}U: $\tau_5 \approx 7 \times 10^8$ years.
Half-life of ^{238}U: $\tau_8 \approx 4.5 \times 10^9$ years.
Latent heat of evaporation of water: $L = 2260\,\mathrm{kJ\,kg^{-1}}$.
Specific heat of water $c = 4200\,\mathrm{J\,kg^{-1}\,K^{-1}}$.
Abundance of ^{235}U in natural uranium today: $R = 0.72\%$. We define abundance as the number of atoms of the isotope, normalized to the number of atoms of the given element.
Average abundance of ^{235}U in the uranium from Oklo's uranium ore today: $R_O = 0.62\%$.
The total amount of uranium in Oklo's mine today: $M = 5 \times 10^8\,\mathrm{kg}$.
The duration of the time period over which Oklo's reactor operated: $T \approx 1 \times 10^5$ year.
Elementary charge: $e = 1.6 \times 10^{-19}\,\mathrm{C}$.
Atomic mass unit: $u = 1.66 \times 10^{-27}\,\mathrm{kg}$.
Avogadro's number: $N_A = 6.02 \times 10^{23}\,\mathrm{mol^{-1}}$.

Note that: (a) the abundance of other isotopes of uranium besides ^{235}U and ^{238}U is negligibly small; (b) ^{235}U is not among the decay products of ^{238}U; and (c) fission channels other than the fission of ^{235}U (e.g., synthesis and fission of plutonium) can be neglected.

i) *(1.5 points)* What was the abundance of ^{235}U in natural uranium when the Oklo's reactor operated?

ii) *(2 points)* What was the average power of the Oklo's reactor?

iii) *(1.5 points)* Qualitatively explain why was Oklo's reactor operating in a stable regime and did not blow up. Water inflow rate varied over time; what happened to the reactor when the water inflow rate increased two times?

iv) *(2 points)* Estimate the total mass of water that flowed into the Oklo's reactor during its operation period.

3. STICKY BALL (4 points) — *Jaan Kalda.*

A glass ball of radius R rests on a flat glass plate. A tiny droplet of water (of surface tension σ) is injected with a syringe so that water forms a small thin neck between the ball and the plate. Both the ball and the plate are perfectly hydrophilic, i.e. the contact angle of water is 0°. Find the increase of the normal force between the plate and the ball caused by the presence of the neck of water.

4. TOTALITY (8 points) — *Taavet Kalda, Jaan Kalda.*

Total solar eclipses are a rare phenomenon which occur when the Moon completely covers the disk of the Sun for some parts of the Earth. This doesn't happen during every solar eclipse because the Moon's apparent size in the sky is sometimes too small to fully cover the Sun, but also because the Moon's shadow usually misses the Earth due its orbital inclination. As a result, total solar eclipses occur on average every 18 months.

Let us consider a total solar eclipse where during the peak, the centre-points of Earth, the Moon and the Sun lie on a line on the same plane as the equator. We measure that right before the total solar eclipse ends at latitude $\lambda = 28.5°$, the totality lasts for $t_0 = 2\,\mathrm{min}$. Earth's radius is $r_e = 6370\,\mathrm{km}$, Moon's radius is $r_m = 1740\,\mathrm{km}$, orbital period of the Moon $T_m = 27.3\,\mathrm{d}$, orbital radius of the Moon $R_m = 384\,000\,\mathrm{km}$. One day on Earth is $T_0 = 24\,\mathrm{hrs}$.

i) *(1.5 points)* For how long is there a place on Earth where the total solar eclipse is observable?

ii) *(1 point)* How many degrees in longitude on Earth does the total solar eclipse cover?

iii) *(1.5 points)* What is the width of the path of totality near the equator?

iv) *(1.5 points)* What is the longest amount of time the total eclipse is visible for a single location on Earth?

v) *(1 point)* For how long is the total eclipse near the location described in iii), at the distance of $a = 50\,\mathrm{km}$ from the centreline of the eclipse path?

vi) *(1.5 points)* Find the average time interval between two total solar eclipses for a given location on Earth by making the following simplifying assumptions:
a) the average width of the full eclipse path is equal to the arithmetic average of its smallest and largest width;
b) typical width of a full eclipse path is half of the average width of the eclipse studied above;
c) typical length of a full eclipse path is equal to the length of the eclipse path studied above if the Earth were not rotating;
d) total solar eclipses occur with equal likelihood anywhere on Earth.

5. STRING AND PENDULUM (10 points) — *Päivo Simson, Jaan Kalda.*

Tools: stand with a clamp; a thread bar that can be fixed horizontally to the stand; one piece of red string and two pieces of orange thread; two wooden blocks for fixing orange threads between the stand's clamp; two steel nuts; tape measure, protractor, permanent marker, one staple of $m = 0.0889\,\mathrm{g}$.

i) *(5 points)* The formula for the oscillation period of a mathematical pendulum is $T = 2\pi\sqrt{l/g}$, where l is the length of the pendulum and g is the acceleration due to gravity, valid only for small oscillations. It is known that, in reality, the period T also depends on the angular amplitude α, which is the maximum deviation angle of the pendulum from the equilibrium position. For small angles $\alpha < \pi/2$, a good approximation is given by the formula $T = 2\pi\sqrt{l/g}(1 + A\alpha^2)$, where α is measured in radians and A is a dimensionless constant. Find the value of the constant A experimentally.

ii) *(5 points)* Find the mass of one meter's worth of red string.

6. CONES (8 points) — *Jaan Kalda, Eppu Leinonen.*

i) *(2 points)* The photo below shows a self-anamorphic drawing — a red heart in green background. The reflection of the red heart in the conical mirror is a reduced green heart. What is the apex angle of the conical mirror? You can take measurements from the photo. The distance where the photo was taken was much larger than the diameter of the red heart.

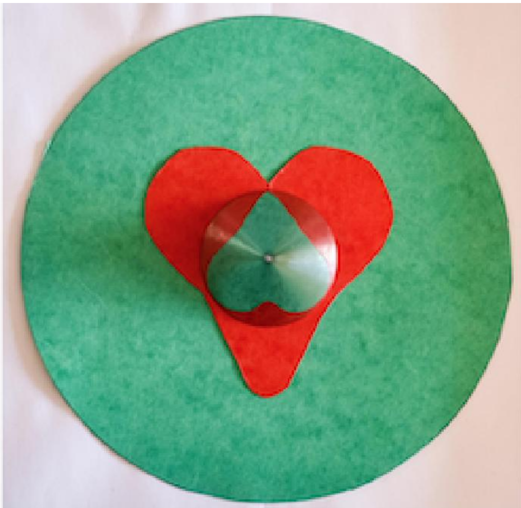


Photo by Erik Mahieu, cf. `https://community.wolfram.com/groups/-/m/t/2027565`.

ii) *(1 point)* A point-like puck of mass m can slide frictionlessly along the internal surface of a cone of half apex angle θ. The gravitational acceleration is g and points along the symmetry axis of the cone at the apex. The puck starts sliding from a point P on the surface of the cone with such a horizontal velocity that it will stay moving at the same fixed height while performing uniform circular motion of radius R. What is its speed v?

iii) *(2.5 points)* Now the puck starts sliding horizontally from the same point P as before, but its initial speed is reduced to $v/2$. What is the smallest distance between the puck and the cone's axis during the subsequent motion?

iv) *(2.5 points)* Now, the cone and the puck are moved to weightlessness. The puck starts again from the point P with the same velocity as in part ii). By how many degrees will the radius vector drawn from the cone's axis to the puck rotate during the subsequent motion? Assume that the cone is infinitely long.

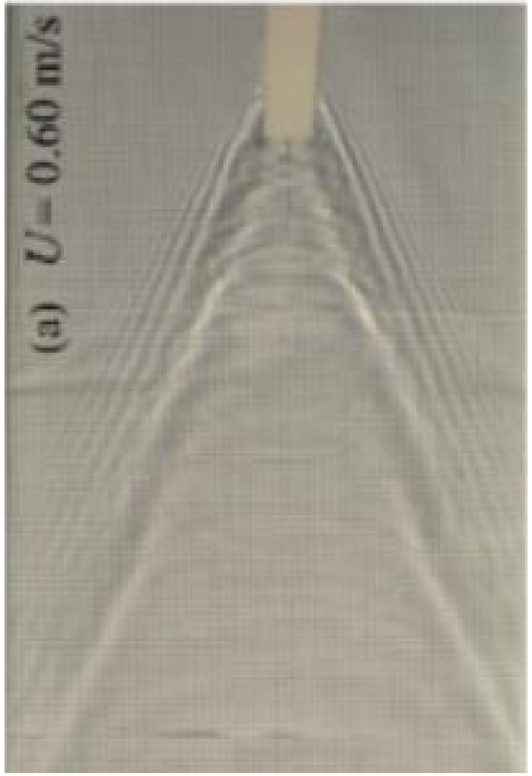
7. WAVES (4 points) — *Janis Huns, Jaan Kalda.*

The dispersion relation (i.e. the dependence of the circular frequency ω on the wave vector $k = \frac{2\pi}{\lambda}$) of capillary-gravity waves is

$$\omega^2 = gk^\alpha + \frac{\sigma}{\rho}k^\beta,$$

where σ denotes the surface tension, $g = 9.81\,\mathrm{m\,s^{-2}}$, and $\rho = 1000\,\mathrm{kg\,m^{-3}}$.

i) *(1 point)* determine the values of the exponents α and β.



F. Moisy, M. Rabaud, PRE 90, 023009 (2014)

ii) *(3 points)* In the image above, we can see how an object moving with a constant speed $U = 60\,\mathrm{cm\,s^{-1}}$ generates a wake — a set of waves of different wavelengths. Pay attention to the short-wavelength waves whose wave crest extends from the object almost up to the edges of the photo: the presence of a very long wavefront testifies that for these particular waves, the phase and group velocities are equal. Determine the surface tension of water. You can take measurements from the photo. Note that while phase speed is the speed of a constant phase of the wave, the group speed $v_g = \frac{d\omega}{dk}$ is the speed of a wave packet (a train of waves).

8. AIRPLANES (7 points) — *Janis Huns, Jaan Kalda.*

Two airplanes pass each other while flying at a constant altitudes. While they have identical airspeeds, their ground speeds are v_1 and v_2, respectively, and the angle between the velocity vectors is α. Based on this knowledge, what is the minimal possible value

i) *(1 point)* of the airspeed of the planes;

ii) *(3 points)* of the wind speed at the altitudes of the planes?

iii) *(3 points)* Answer the question ii) if now $v_1 = v_2 = v$, but it is known that the airspeed of one of the planes is two times bigger than that of the other.

9. TRIANGLE (5 points) — *Jaan Kalda.*

Three identical small iron balls were initially arranged in an equilateral triangle formation, connected by massless non-stretchable threads. Upon being thrown into the air, the system experienced the following conditions: (a) all threads were taut initially; (b) all balls possessed different velocities; (c) all velocities were confined to the plane of the triangle as the system underwent free fall within Earth's gravitational field. At a certain moment $t = 0$, two threads ruptured, leaving two balls tethered together while the third ball separated from the rest of the system. The accompanying diagram depicts the positions of all three balls and the remaining thread within the plane of their initial arrangement at a later moment of time $t = T$ when all the balls were still continuing their free fall. To answer the questions below, you can take measurements from the figure.

i) *(4 points)* By how many degrees did the remaining thread rotate during the time period from $t = 0$ to $t = T$?

ii) *(1 point)* Was the rotation clockwise or counterclockwise?

10. KITCHEN PHYSICS (12 points) — *Eero Uustalu, Jaan Kalda.*

Tools: aluminum foil laminated brown paper bag*; a kitchen sponge with abrasive green side (for removing the varnish from the aluminum surface of the aluminum laminated bag to achieve good electrical contact), two sheets of aluminum foil*, two sheets of cushioning foam*, a square-shaped piece of wooden laminated plate, a sheet of kitchen plastic wrap*, permanent marker, scissors, micrometer, ruler, multimeter with crocodile clips. You can ask for a replacement if items marked with a star become damaged. The resistivity of aluminum $\rho = 2.7 \times 10^{-8}\,\Omega\cdot\mathrm{m}$; permittivity of vacuum $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$.

i) *(6 points)* Find the thickness of the aluminium foil coating of the brown paper bag.

ii) *(6 points)* Find the relative permittivity of the plastic wrap.