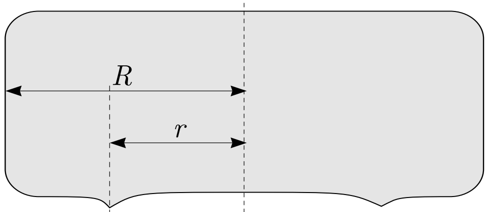
Nordic-Baltic Physics Olympiad 2023

1. Curling (8 points) — *Oskar Vallhagen.*

In the sport of curling, participants take turns sliding near-cylindrical stones across an ice court towards a target, trying to get their stones as close to the target as possible after using a set of stones. A vertical cross section of a stone is depicted below, showing that the stone is in contact with the ice on a thin ring of radius r. The full radius of the stone is R, the mass of the stone is m and the coefficient of friction with the ice is μ.



Consider the case when the stone is released at a speed v_0 with the aim of knocking out an opponent's stone at a distance s.

i) *(1 point)* Give an expression for the sliding speed v_s as a function of the time t since the stone was released until the stone hits the opponent's stone.

ii) *(1 point)* What is the sliding speed v_{hit} just before the stone hits the opponent's stone?

Now the stone is given a small rotation at the initial angular speed ω_0 (this can be done to alter the deflection angle of the stone when it hits the opponent's stone). Assume that the rotation speed ω remains small throughout the sliding motion: $\omega r \ll v_s$. Keep only the main non-vanishing terms in your calculations, i.e. among the terms with a factor $(\omega r/v_s)^n$, keep only the term with the smallest n. Depending on your approach you may use the following approximations for $x \ll 1$: $(1+x)^\alpha \approx 1+\alpha x+\frac{1}{2}\alpha(\alpha-1)x^2$, $\sin(\alpha+x) \approx \sin\alpha + x\cos\alpha$, $\cos x \approx 1-x^2/2$. You may also need the integral $\int(at+b)^{-1}dt = a^{-1}\ln|at+b|+C$.

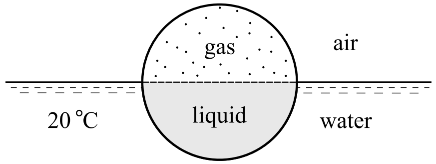
iii) *(2 points)* By how much does the friction force on the stone change due to its rotation? Express your answer in terms of the current angular speed ω and sliding speed v_s.

iv) *(2 points)* Give an expression for the torque T exerted on the stone.

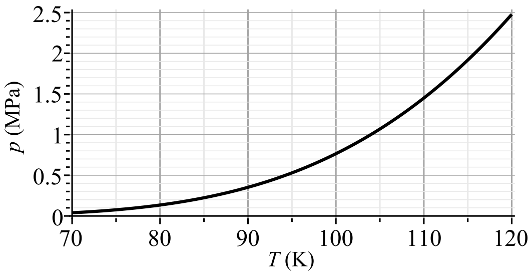
v) *(2 points)* What is the angular speed of the stone just before it hits the opponent's stone?

2. Nitrogen explosion (8 points) — *Päivo Simson.*

A half-sphere of radius $r = 0.1\,\mathrm{m}$ is filled with liquid nitrogen at the boiling point temperature of $T_1 = 77.4\,\mathrm{K}$ ($-195.8\,^\circ\mathrm{C}$). The other half is then firmly sealed onto the first, creating a sphere containing liquid nitrogen and nitrogen gas, each occupying one-half of the volume. The sphere is immediately thrown into $T_w = 20\,^\circ\mathrm{C}$ temperature water, where it floats exactly as shown in the figure below. After some time, it explodes.



The sphere is made of PCTFE plastic of density $\rho_p = 2130\,\mathrm{kg\,m^{-3}}$, maximum tensile strength $\sigma = 3.4\times10^7\,\mathrm{N\,m^{-2}}$ (above this stress, the plastic will break) and thermal conductivity $k = 0.84\,\mathrm{W\,m^{-1}\,K^{-1}}$. For liquid nitrogen, under the conditions considered here, the latent heat of vaporization is $\lambda = 2.0\times10^5\,\mathrm{J\,kg^{-1}}$, specific heat $c_v = 2000\,\mathrm{J\,kg^{-1}\,K^{-1}}$ and density $\rho_n = 808\,\mathrm{kg\,m^{-3}}$. Molar mass $\mathrm{M(N_2)} = 28\,\mathrm{g\,mol^{-1}}$. The ideal gas constant $R = 8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$. Temperature dependence of saturated vapor pressure of nitrogen is shown below.



i) *(1.5 points)* What is the wall thickness d of the sphere?

ii) *(1.5 points)* What is the pressure p_2 inside the sphere right before it explodes? The outside pressure is $p_a = 1.0\times10^5\,\mathrm{Pa}$.

iii) *(1.5 points)* What is the temperature T_2 of liquid nitrogen right before the explosion?

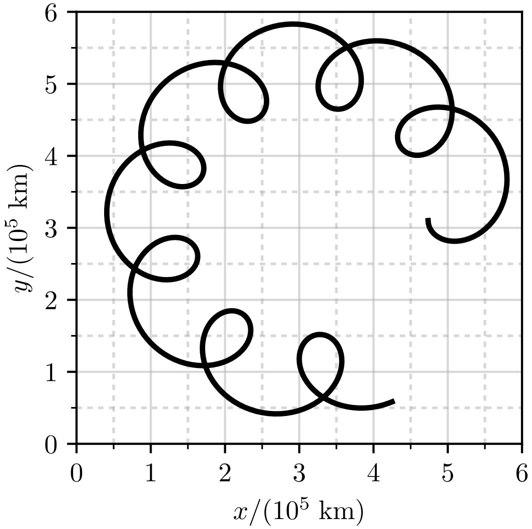
iv) *(1.5 points)* Calculate the mass of nitrogen that evaporates inside the sphere before the explosion.

v) *(2 points)* Estimate the time it will take for the sphere to explode. The heat capacity of the plastic and the heat flux through the upper half of the sphere can be neglected.

3. Wobble (8 points) — *Taavet Kalda.*

We investigate the so-called astrometric wobble method for detecting exoplanets. The method relies on the ability to measure the change in position in the sky that the host star experiences due to the gravitational influence of its orbiting planets. This method has become feasible in recent years due to the availability of more precise instruments.

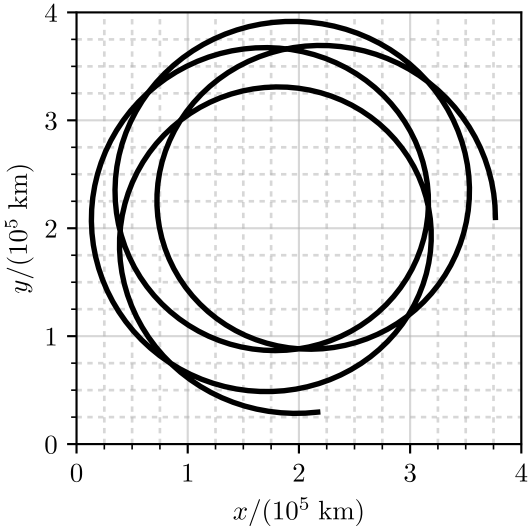
i) *(2.5 points)* Consider a system consisting of a host star, an inner planet A, and an outer planet B. Below is a measurement of the trajectory of the centre of the star in the plane perpendicular to the line of sight, measured over a period of $t = 10\,\mathrm{yr}$. In all of the subsequent parts, you may assume that both planets orbit in circular orbits in the plane of the diagram. What are the orbital periods T_A and T_B of the two planets?



ii) *(2.5 points)* Based on direct imaging of planet A in the infrared, the orbital radius of planet A is measured to be $a_A = 1.5\,\mathrm{AU} = 2.2\times10^8\,\mathrm{km}$. What is the mass M of the host star, and the mass m_A of planet A? The gravitational constant is $G = 6.67\times10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$.

iii) *(1 point)* What is the mass m_B of planet B?

iv) *(2 points)* Now consider a similar $t = 10\,\mathrm{yr}$ measurement of a different system shown below, also consisting of a host star and two planets A and B (where A is the inner planet). Similarly to before, find the mass M of the host star, and the masses m_A, m_B of the planets in the new system. As before, direct imaging yields that the orbital radius of planet A is $a_A = 1.3\,\mathrm{AU} = 2.0\times10^8\,\mathrm{km}$.



4. Black box (12 points) — *Jaan Kalda, Eero Uustalu.*

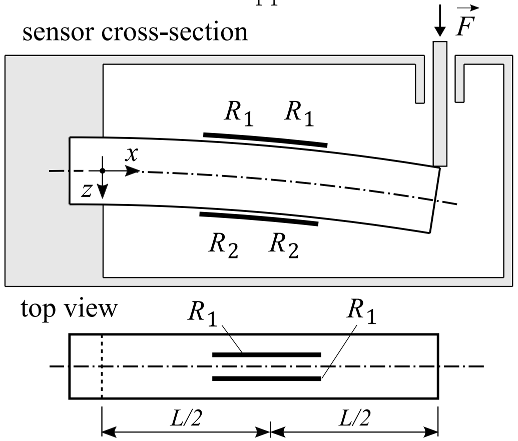
Tools: a black box with two output terminals, a multimeter, a stopwatch, wires.

The resistance of the multimeter is as follows: when used as microammeter — $103.2\,\Omega$; when used as a milliammeter — $4.2\,\Omega$; when used as a voltmeter — more than $10\,\mathrm{M\Omega}$. When used as a voltmeter, the multimeter will automatically determine the optimum range, but this will slow it down; to make it faster, press the "Range" button to fix it to the current range.

Which four components (apart from the wires; there can be more than one of the same type) are in the black box? What are the values of the quantities that characterise these components and how could they be connected? Document and tabulate all your measurements and plot them graphically where appropriate.

5. Force sensor (5 points) — *Päivo Simson.*

A force sensor is constructed from a flexible beam of working length L and height h to which four identical electrically resistive wires of length $l \ll L$ are attached, as shown in the figure below. If a force F is applied to the end of the beam, it will bend the beam and stretch the upper wires and compress the lower ones, causing changes in the electrical resistance. Using the Wheatstone bridge method, these changes can be converted into a voltmeter reading, allowing for the measurement of the applied force F.

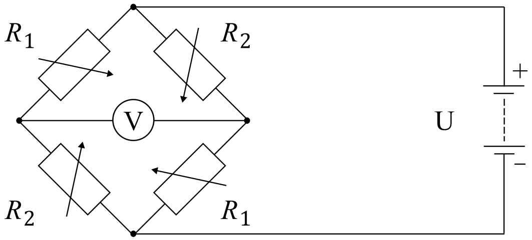


In what follows, assume that the deflection of the beam is very small.

i) *(2 points)* The bending moment $M(x)$ (i.e. the torque with which one fictitious half of the beam affects the other) is inversely proportional to the curvature radius $r(x)$ of the center line of the beam: $M = EI/r$, where E is Young's modulus and I is a constant depending on the geometry of the of the beam's cross-section (both are known). Find the elongation Δl of the upper wire when a force F is applied to the sensor. Assume that the wires are placed in the middle of the beam.

ii) *(1 point)* Find the resistances R_1 and R_2 when Δl from the previous section is known and the initial resistance of the wires is R_0. Assume that the volume of the wire remains constant during the deformation.

iii) *(2 points)* The resistors are arranged in a Wheatstone bridge configuration shown in the figure below, where U is the known battery voltage. Find the relationship between the measured voltage V and the force F.

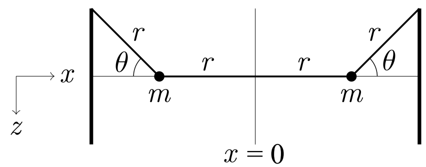


6. String-coupled masses (5 points) — *Aleksi Kononen.*

Two small masses m are connected and hung between walls by weightless strings as pictured below. The acceleration due to gravity is g. All oscillatory motion is assumed to have a small amplitude and the system is always in one of its normal modes (i.e. the oscillations are sinusoidal).

i) *(2 points)* Find ω_1, the angular frequency of in-phase oscillations (by which the oscillation phases of the both masses are always equal), perpendicular to the plane of the figure.

ii) *(3 points)* Find ω_2, the angular frequency of anti-phase oscillations (by which the oscillation phases of the both masses are always opposite), in terms of ω_1.



7. A stack of papers (8 points) — *Taavet Kalda and Jaan Kalda.*

There is a pack of $N \gg 1$ identical sheets of papers lying on an infinite horizontal table. The coefficient of friction between the surface of the table, and between two sheets of paper are both equal to μ. Each sheet has dimensions $L \times W$, with $L > W$. Sandra is trying to fetch the bottom-most sheet by pulling from a shorter edge of it with a constant velocity u (while all the sheets lie almost exactly on top of each other, she managed to get hold of an edge of the bottom-most sheet).

i) *(2 points)* Sketch a qualitative graph of how the acceleration a of the pack (excluding the bottom-most sheet) depends on time when
(a) the speed u is very small.
(b) the speed u is very big.

ii) *(3 points)* What is the minimal speed $u_{\min}$ by which it is possible to pull the bottom-most sheet out (i.e. separating it completely from the remaining pack)?

iii) *(1 point)* Assuming $u > u_{\min}$, what is the speed of the remaining pack at the moment when the bottom-most sheet gets out of the pack (i.e. there is no longer overlapping areas)?

iv) *(2 points)* Considering still $u > u_{\min}$, what is the minimal distance l between the pack of papers and the edge of the table such that the pack would not slide over the edge of the table? (l is the maximal allowed travel distance of the pack.)

8. Connected charges (8 points) — *Jaan Kalda.*

In the region $0 < x < L$, there is an electric field $\vec{E} = E_0\hat{x}$, where $\hat{x}$ denotes the unit vector parallel to the x-axis. Two small balls, each of mass m and carrying charge q ($qE_0 > 0$) are connected with a weightless non-stretchable string of length l. Initially, at the moment of time $t = 0$, the string is taut, the velocity of the both balls is $\vec{v} = v_0\hat{x}$, one of the balls, the ball A, is at $x = 0$ while the other ball, the ball B is at $x < 0$. The electric field created by the balls can be neglected, and it can be assumed that v_0 is very small (much smaller than $\sqrt{ELq/m}$).

i) *(2 points)* Consider the case when the string is parallel to the x-axis, and $l = L$ Sketch the dependence of the velocity of the both balls as a function of time. Will the balls collide? If yes then when?

ii) *(2 points)* Now, at $t = 0$, the string forms an angle of 45° with the x-axis, $l = 1.2291L$. By this string length, at the moment $t = T$ when the ball A reaches $x = L$, the string is parallel to the x-axis. Find T.

iii) *(2 points)* Under the assumptions of the previous task, what is the speed of the ball A at the moment $t = T$?

iv) *(2 points)* Now, at $t = 0$, the string forms a very small angle ϕ with the x-axis. Similarly to the previous two tasks, at the moment $t = T'$ when the ball A reaches $x = L$, the string is parallel to the x-axis. Find the string length l assuming that $l > L$. *The answer to this point should contain only L and numerical constants.*

9. Surface tension (10 points) — *Jaan Kalda, Eero Uustalu.*

Tools: a syringe, a small glass plate, a support for holding the glass plate horizontally at an adjustable height, a cup with water (coefficient of refraction of water $n_w = 1.33$), a caliper, a sheet of graph paper. Your working room has ceiling lights at an approximate height of 3 m. The glass plate holder is a short piece of plastic pipe with a nut; put the glass plate on top of the nut and turn the nut to adjust the height. By turning the nut, one can change the holder height only by the thickness of the nut; there are also spare nuts that can be stacked to increase the total height of the pipe-nuts system, and a cap that fits into the pipe — use it if there is a need to make the distance between the glass plate and the surface beneath it smaller than the height of the pipe. NB! Hold the glass plate only from its matte part and do not touch the glossy (transparent) part as fingerprints will affect the value of the contact angle. If you accidentally do touch, ask organisers to clean the glass surface.

i) *(3 points)* Put a small drop of water onto the glass plate; this will form a plano-convex lens. Determine the focal length of this lens and measure its diameter.

ii) *(2 points)* Calculate the curvature radius of the water surface and the water-glass contact angle α. The contact angle is defined as the angle under which the water surface (the air-water interface) meets the surface of the glass plate.

iii) *(3 points)* Now increase the amount of water on the glass plate so that it covers a big part of the glass plate so that its top surface becomes almost flat. Determine the thickness of the water layer.

iv) *(2 points)* Calculate the surface tension of water σ. Hint: a given amount of water takes a shape which minimises its total potential energy. Use reasonable approximations. Keep in mind that the glass-water interface makes also a certain contribution U_{gw} to the full surface energy, the magnitude of which is related to the contact angle: $U_{gw} = -\sigma\cos\alpha A$, where A denotes the surface area of the glass-water interface.