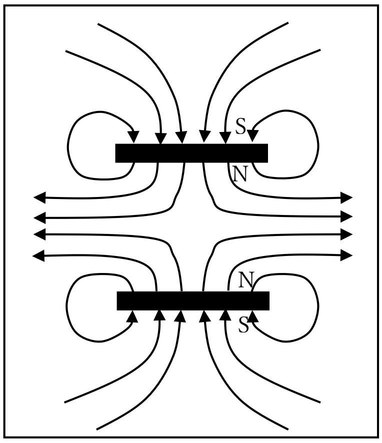
Mass Measurement (10 points)

Write down the numbers 0 to 9 in the following table:

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

Part A: Hooke's law and electromagnetic forces (2.4 points)

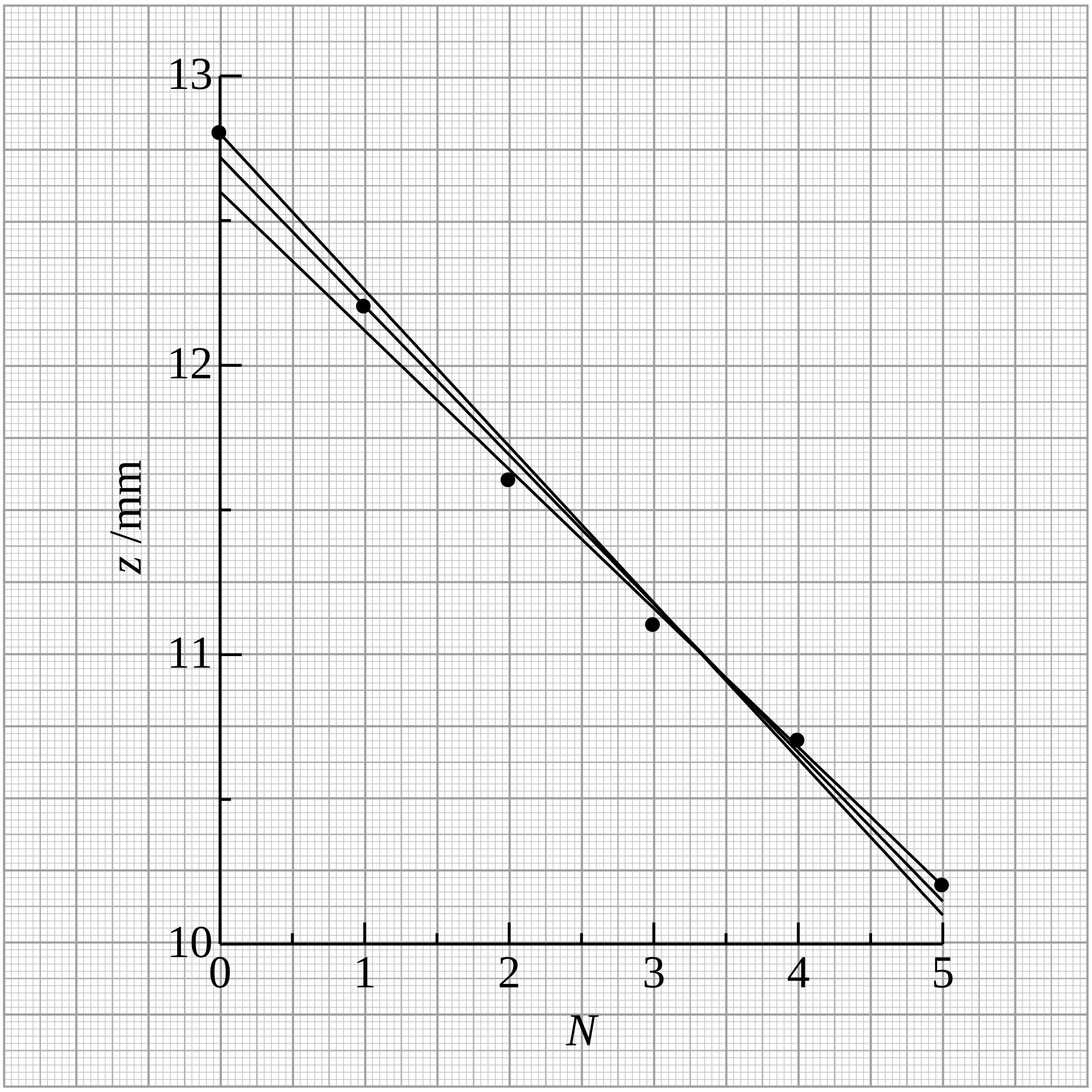
A.1 (0.4 pt)



A.2 (0.6 pt)

N	z /mm	I /A
0	12.8	0
1	12.2	0.103
2	11.6	0.213
3	11.1	0.323
4	10.7	0.423
5	10.2	0.524

A.3 (0.7 pt)



The slope and uncertainty are read from the lines plotted on the graph.

$$a = \frac{\Delta z}{\Delta N} = \frac{10.15 - 12.70}{5} = -0.51$$

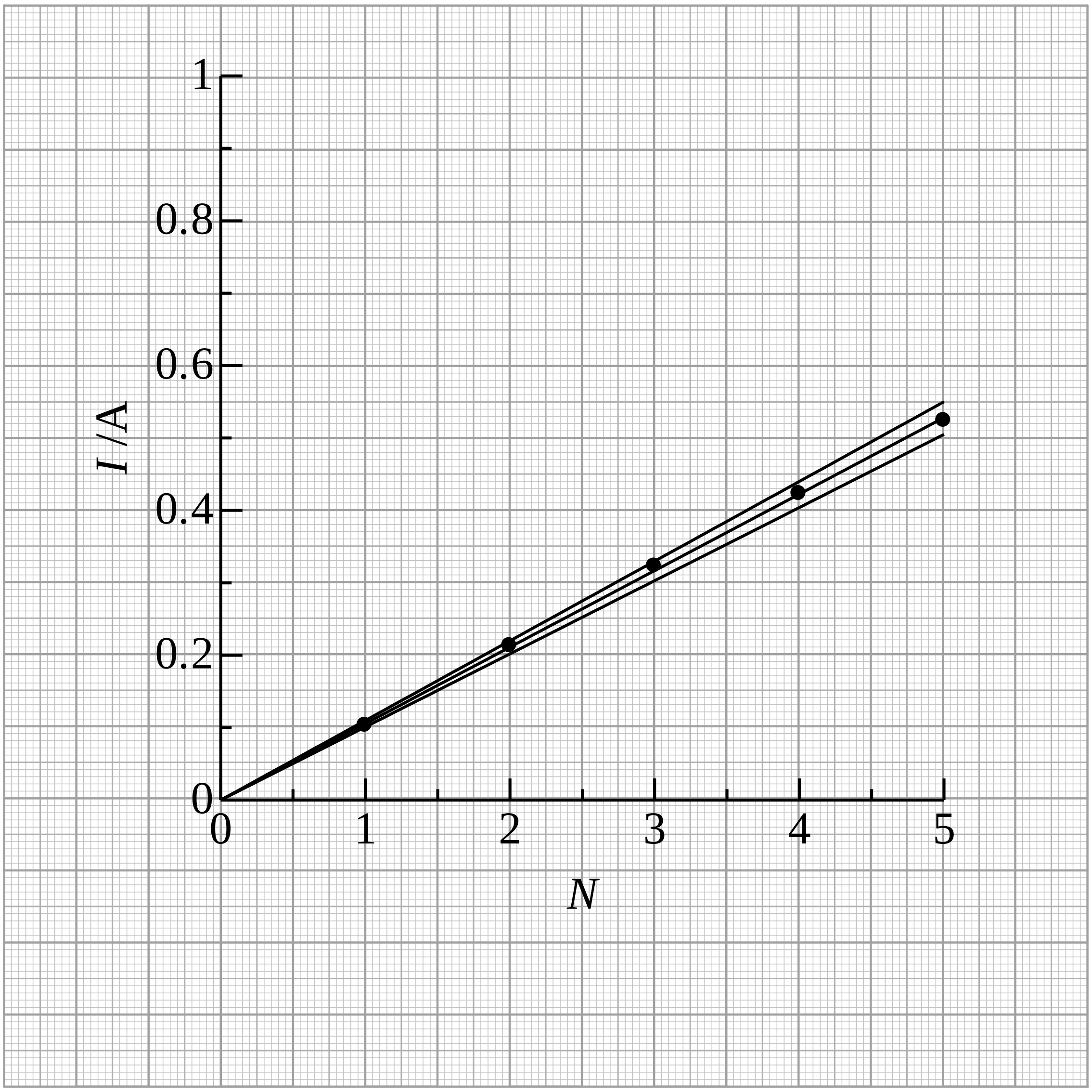
$$a_+ = \frac{10.20 - 12.60}{5} = -0.48$$

$$a_- = \frac{10.10 - 12.80}{5} = -0.54$$

$$\Delta a = \frac{-0.48 - (-0.54)}{2} = 0.03$$

$$a = -0.51 \pm 0.03 \text{ mm}$$

A.4 (0.7 pt)



The slope and uncertainty are read from the lines plotted on the graph.

$$b = \frac{I}{N} = \frac{0.53}{5} = 0.106$$

$$b_+ = \frac{0.55}{5} = 0.110$$

$$b_- = \frac{0.505}{5} = 0.101$$

$$\Delta b = \frac{0.110 - 0.101}{2} = 0.005$$

$$b = \ 0.106 \pm 0.005 \text{ A}$$

Part B: Induced electromotive force (3.0 points)

B.1 (0.2 pt)

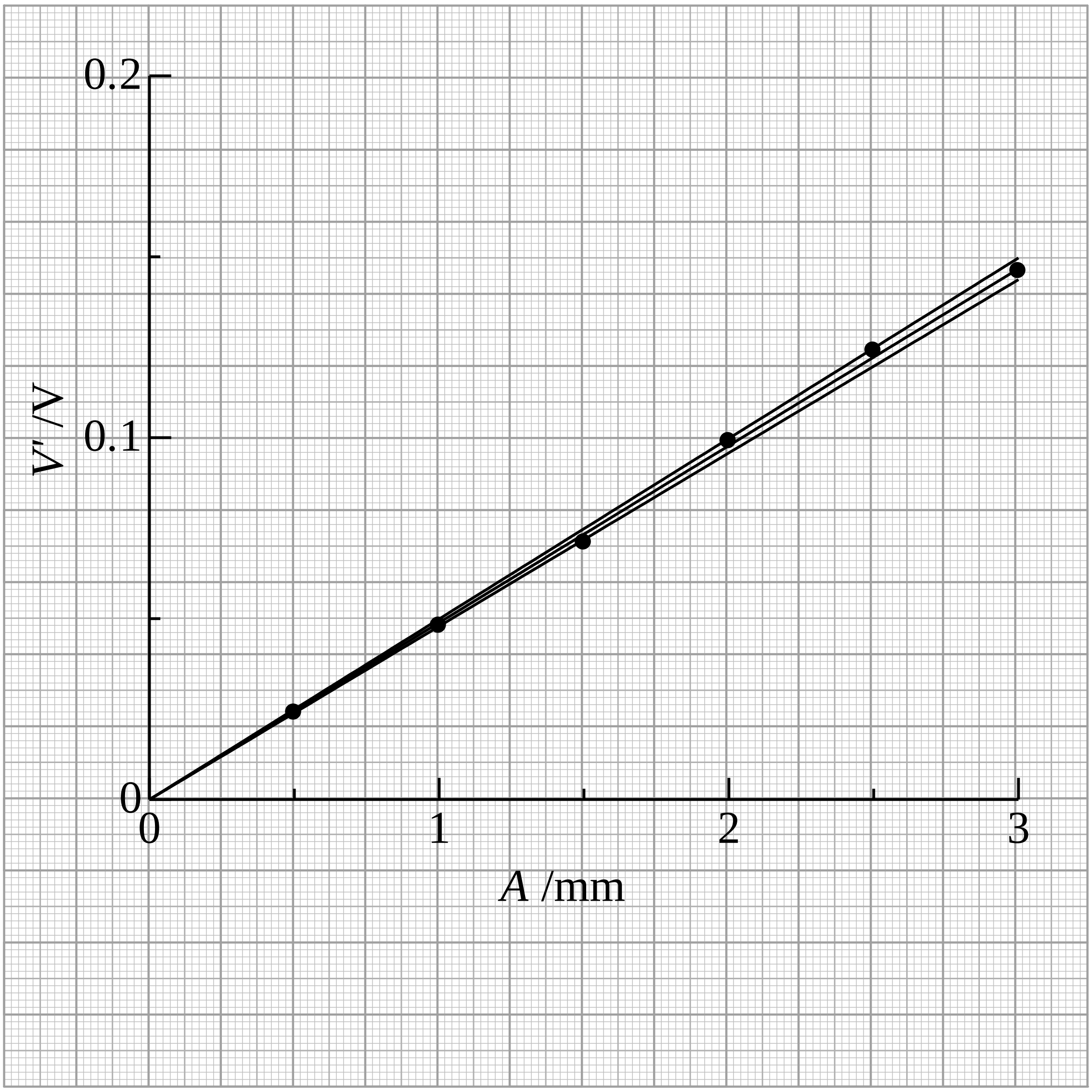
$V = 2\pi f ABL$

B.2 (0.5 pt)

$f_{\mathrm{B}} = 15.85\ \mathrm{Hz}$

A /mm	V' /V
0.5	0.024
1.0	0.048
1.5	0.071
2.0	0.099
2.5	0.124
3.0	0.146

B.3 (0.7 pt)



The slope and uncertainty are read from the lines plotted on the graph.

$$c = \frac{V'}{N} = \frac{0.147}{5} = 0.049$$

$$c_+ = \frac{0.150}{5} = 0.050$$

$$c_- = \frac{0.144}{5} = 0.048$$

$$\Delta c = \frac{0.050 - 0.048}{2} = 0.001$$

$$c = \ 0.049 \pm 0.001 \text{ V/mm}$$

B.4 (0.4 pt)

$$BL = \frac{V}{2\pi A f_\mathrm{B}} = \frac{\sqrt{2}V'}{2\pi A f_\mathrm{B}} = \frac{\sqrt{2}c}{2\pi f_\mathrm{B}} = \frac{\sqrt{2}\times 0.049}{2\pi \times 15.85} = 0.000696\ \mathrm{Vs/mm} = 0.696\ \mathrm{Vs/m}$$

$$\Delta(BL) = BL \cdot \frac{\Delta c}{c} = 0.696 \times \frac{0.001}{0.049} = 0.014\ \mathrm{Vs/m}$$

$$BL = \ 0.696 \pm 0.014\ \mathrm{Vs/m}$$

B.5 (1.2 pt)

$$m = \frac{mg}{BL}\cdot\frac{BL}{g} = \frac{I}{N}\cdot\frac{V}{2\pi A f_\mathrm{B}}\cdot\frac{1}{g} = b\frac{\sqrt{2}c}{2\pi g f_\mathrm{B}} = 0.106 \times \frac{\sqrt{2}\times 0.049}{2\pi \times 9.80 \times 15.85} = 0.0075\ \mathrm{kg} = 7.5\ \mathrm{g}$$

The principle of the Kibble balance (watt balance)

Mechanical power: $Fv = Nmg \cdot 2\pi A f_\mathrm{B}$

Electrical power: VI

$$Fv = VI$$

$$\Delta m = m \cdot \sqrt{(\frac{\Delta b}{b})^2 + (\frac{\Delta c}{c})^2} = 0.4\ \mathrm{g}$$

$$m = \ 7.5 \pm 0.4\ \mathrm{g}$$

$$k = -\frac{mg}{a} = -\frac{7.5 \times 9.80}{-0.51} = 144\ \mathrm{N/m}$$

$$\Delta k = k \cdot \sqrt{(\frac{\Delta a}{a})^2 + (\frac{\Delta m}{m})^2} = 11\ \mathrm{N/m}$$

$$k = \ 144 \pm 11\ \mathrm{N/m}$$

Part C: Mass dependence of resonant frequency (2.3 points)

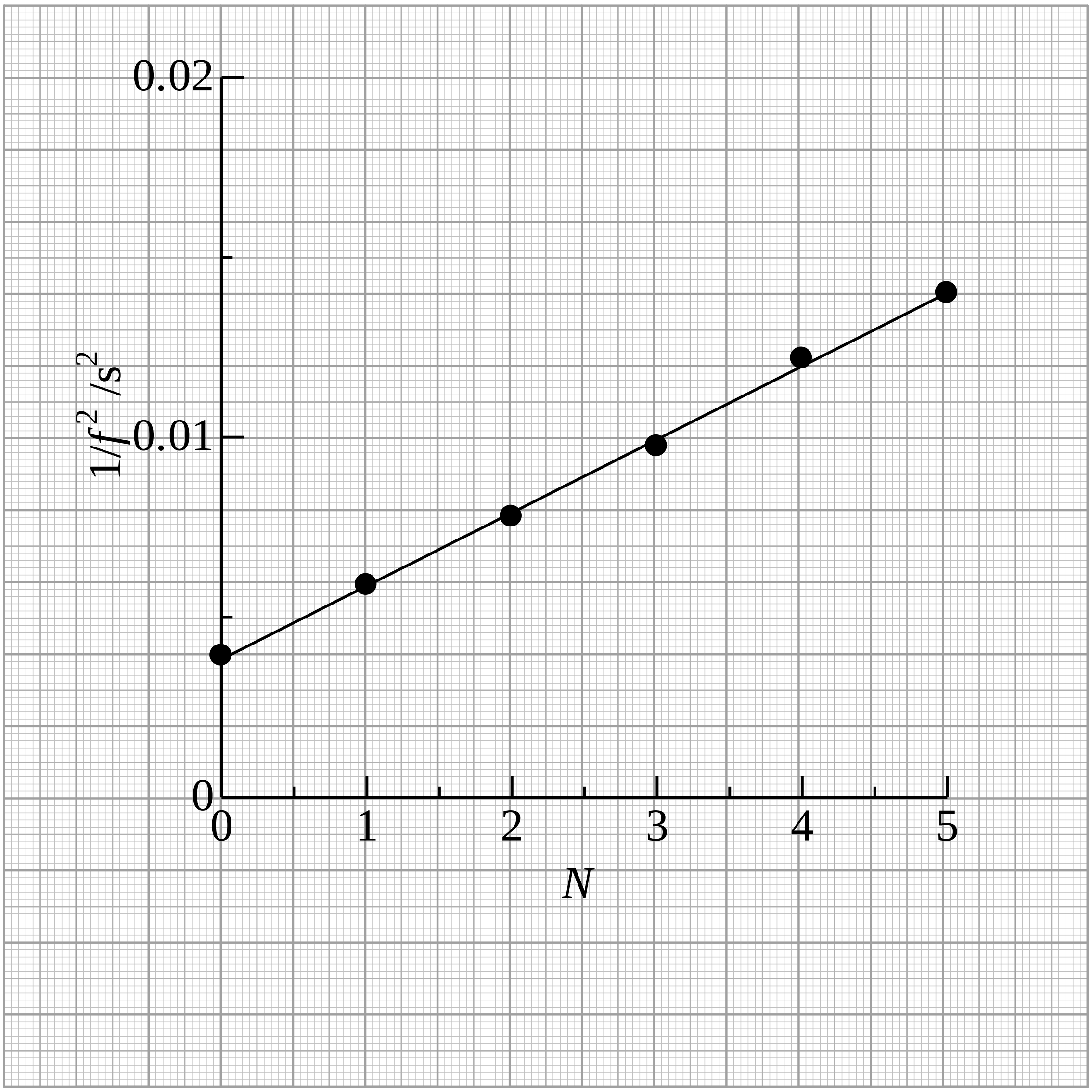
C.1 (0.2 pt)

$$f = \frac{1}{2\pi}\sqrt{\frac{k'}{M + Nm}}$$

C.2 (0.5 pt)

N	f /Hz	$1/f^2$ /s^2		
0	15.96	0.003926		
1	13.03	0.005390		
2	11.33	0.007790		
3	10.13	0.009745		
4	9.06	0.01218		
5	8.45	0.01401		

C.3 (1.0 pt)



The additional quantities $1/f^2$ are calculated in Table C.2. Then, $\frac{M}{k'}$ and $\frac{m}{k'}$ are obtained from the graph using the equation $\frac{1}{f^2} = (2\pi)^2(\frac{M}{k'} + \frac{m}{k'}N)$.

$$\frac{M}{k'} = \frac{0.0039}{(2\pi)^2} = 9.88 \times 10^{-5}\ \mathrm{s}^2$$

$$\frac{m}{k'} = \frac{(0.0140 - 0.0039)/5}{(2\pi)^2} = 5.12 \times 10^{-5}\ \mathrm{s}^2$$

C.4 (0.6 pt)

$$\frac{M}{m} = \frac{M/k'}{m/k'} = \frac{9.88}{5.12} = 1.93$$

$$\frac{M}{m} = 1.93$$

$$M = \frac{M}{m} \cdot m = 1.93 \times 0.0075 = 0.0145 \text{ kg}$$

$$M = 14.5 \text{ g}$$

$$k' = \frac{M}{M/k'} = \frac{0.0145}{9.88 \times 10^{-5}} = 147 \text{ N/m}$$

$$k' = 147 \text{ N/m}$$

Part D: Resonance characteristics (2.3 points)

D.1 (0.4 pt)

$$V'_{AC} = 0.157\ \mathrm{V}$$

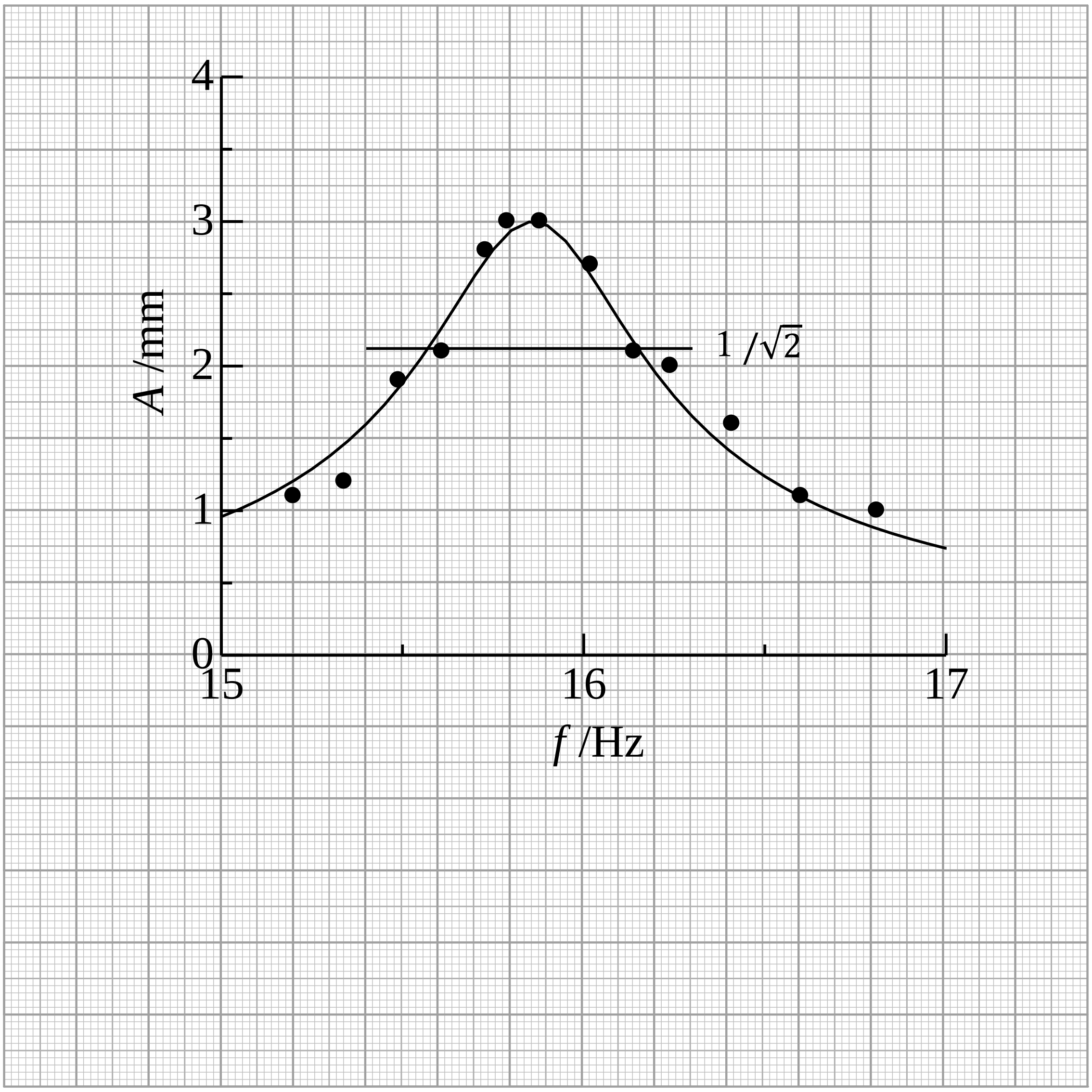
$$F_{AC} = BLI_{AC} = BL \times 0.106 \times \sqrt{2}V'_{AC} = 0.696 \times 0.106 \times \sqrt{2} \times 0.157 = 0.0164\ \mathrm{N}$$

$$F_{AC} = 0.0164\ \mathrm{N}$$

D.2 (0.9 pt)

f /Hz	A /mm	$(f-f_0)^2$ /Hz^2	$1/A^2$ /mm^{-2}
15.88	3.0	0.0064	0.111
15.79	3.0	0.0289	0.111
15.73	2.8	0.0529	0.128
15.61	2.1	0.1225	0.227
15.49	1.9	0.2209	0.277
15.34	1.2	0.3844	0.694
15.20	1.1	0.5776	0.826
16.02	2.7	0.0036	0.137
16.14	2.1	0.0324	0.227
16.24	2.0	0.0784	0.250
16.41	1.6	0.2025	0.391
16.60	1.1	0.4096	0.826
16.81	1.0	0.7225	1.000

D.2 (cont.)



D.3 (1.0 pt)

Reading from the graph D.2

$f_0 = 15.83$ Hz

$A(f_0) = 3.0$ mm

$$\Delta f = \frac{15.14 - 15.56}{2} = 0.29 \text{ Hz}$$

Calculaton using Eq.(4)

$$M = \frac{F_{\rm AC}}{8\pi^2 f_0 \Delta f A(f_0)} = \frac{0.0164}{8\pi^2 \times 15.83 \times 0.29 \times 0.003} = 0.0151 \text{ kg}$$

$M = 15.1$ g

An alternative way to find M

$(f - f_0)^2$ and $1/A^2$ are calculated in Table D.2 to use the linear relationship

$$\frac{1}{A^2} = (\frac{8\pi^2 M f_0}{F_{\rm AC}})^2 \cdot [(f - f_0)^2 + (\Delta f)^2]\,.$$

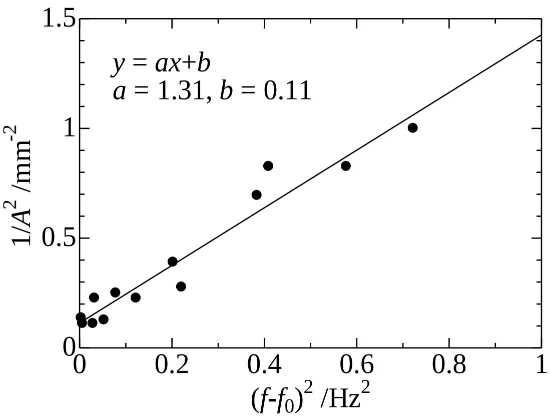
$f_0 = 15.96$ Hz obtained in C.2 is used.

The slope is obtained from the graph of the additional quantities or the calculation

$$(\frac{8\pi^2 M f_0}{F_{\rm AC}})^2 = 1.31 \text{ mm}^{-2}\text{Hz}^{-2} = 1.31 \times 10^6 \text{ m}^{-2}\text{Hz}^{-2}.$$

$$M = \sqrt{1.31 \times 10^6} \times \frac{F_{\rm AC}}{8\pi^2 f_0} = \sqrt{1.31 \times 10^6} \times \frac{0.0164}{8\pi^2 \times 15.96} = 0.0149 \text{ kg}$$

$M = 14.9$ g



Thickness Measurements Using Birefringence (10 points)

Values in black (blue) are typical (acceptable).

Part A. Measurement System Setup (2.3 points)

A.1 (0.3 pt)

$\lambda = 684$ nm

$\theta = 20.0°$

A.2 (0.2 pt)

$\theta = -30.0°,\ 70.0°$

A.3 (0.8 pt)

$\theta = -28.0°$ $\quad (-28.9° \le \theta \le -27.7° \quad \text{or} \quad 67.7° \le \theta \le 68.9°)$

$\lambda_{\text{Peak}} = 458$ nm

(450 nm $\le \lambda_{\text{Peak}} \le 460$ nm; λ_{Peak} and θ must be consistent with Eq. (7) or (8).)

$\alpha = 40.0°$

A.4 (0.3 pt)

$\varphi_{\perp} = 90°$ $\quad (85° \le \varphi_{\perp} \le 95° \quad \text{or} \quad 265° \le \varphi_{\perp} \le 275°)$

$\varphi_{\parallel} = 0°$ $\quad (\varphi_{\parallel} = \varphi_{\perp} + 90° \ \text{or}\ \varphi_{\perp} - 90°)$

A.5 (0.2 pt)

$I_{\text{Offset}\perp} = 0.005\ \text{V}$ $\quad (I_{\text{Offset}\perp} \leq 0.010\ \text{V})$

$I_{\text{Offset}\parallel} = 0.010\ \text{V}$ $\quad (I_{\text{Offset}\parallel} \leq 0.020\ \text{V})$

A.6 (0.5 pt)

$I_{\perp} = 0.001\ \text{V}$ $\quad (I_{\perp} \leq 0.003\ \text{V})$

$I_{\parallel} = 0.160\ \text{V}$ $\quad (I_{\parallel} \geq 0.100\ \text{V})$

Part B. Measurement of transmitted light intensities (4.7 points)

B.1 (2.0 pt)

θ_{Stage}/degree	θ/degree	λ/nm	$I_\perp$/mV	$I_\parallel$/mV	I_{Total}/mV	I_{Norm}
30.5	-31	430.5	13	26	39	0.333
30	-30.5	435.1	38	31	69	0.551
29.5	-30	439.7	83	27	110	0.755
29	-29.5	444.2	166	17	183	0.907
28.5	-29	448.8	244	21	265	0.921
28	-28.5	453.3	280	80	360	0.778
27.5	-28	457.7	267	159	426	0.627
27	-27.5	462.1	188	216	404	0.465
26.5	-27	466.5	73	223	296	0.247
26	-26.5	470.9	17	197	214	0.079
25.5	-26	475.2	12	162	174	0.069
25	-25.5	479.5	19	121	140	0.136
24.5	-25	483.7	34	71	105	0.324
24	-24.5	487.9	48	43	91	0.527
23.5	-24	492.1	61	22	83	0.735
23	-23.5	496.2	72	10	82	0.878
22.5	-23	500.3	83	4	87	0.954
22	-22.5	504.3	94	8	102	0.922
21.5	-22	508.3	97	19	116	0.836

B.1 (2.0 pt)

(Continued)

θ_{Stage}/degree	θ/degree	λ/nm	$I_\perp$/mV	$I_\parallel$/mV	I_{Total}/mV	I_{Norm}
21	-21.5	512.3	92	37	129	0.713
20.5	-21	516.3	77	68	145	0.531
20	-20.5	520.1	61	90	151	0.404
19.5	-20	524.0	35	130	165	0.212
19	-19.5	527.8	18	153	171	0.105
18.5	-19	531.6	8	166	174	0.046
18	-18.5	535.3	8	167	175	0.046
17.5	-18	539.0	14	158	172	0.081
17	-17.5	542.7	32	141	173	0.185
16.5	-17	546.3	47	127	174	0.270
16	-16.5	549.9	73	99	172	0.424
15.5	-16	553.4	93	76	169	0.550
15	-15.5	556.9	112	55	167	0.671
14.5	-15	560.3	130	34	164	0.793
14	-14.5	563.7	141	20	161	0.876
13.5	-14	567.1	147	10	157	0.936
13	-13.5	570.4	148	6	154	0.961
12.5	-13	573.7	146	6	152	0.961
12	-12.5	576.9	138	10	148	0.932

B.1 (2.0 pt)

(Continued)

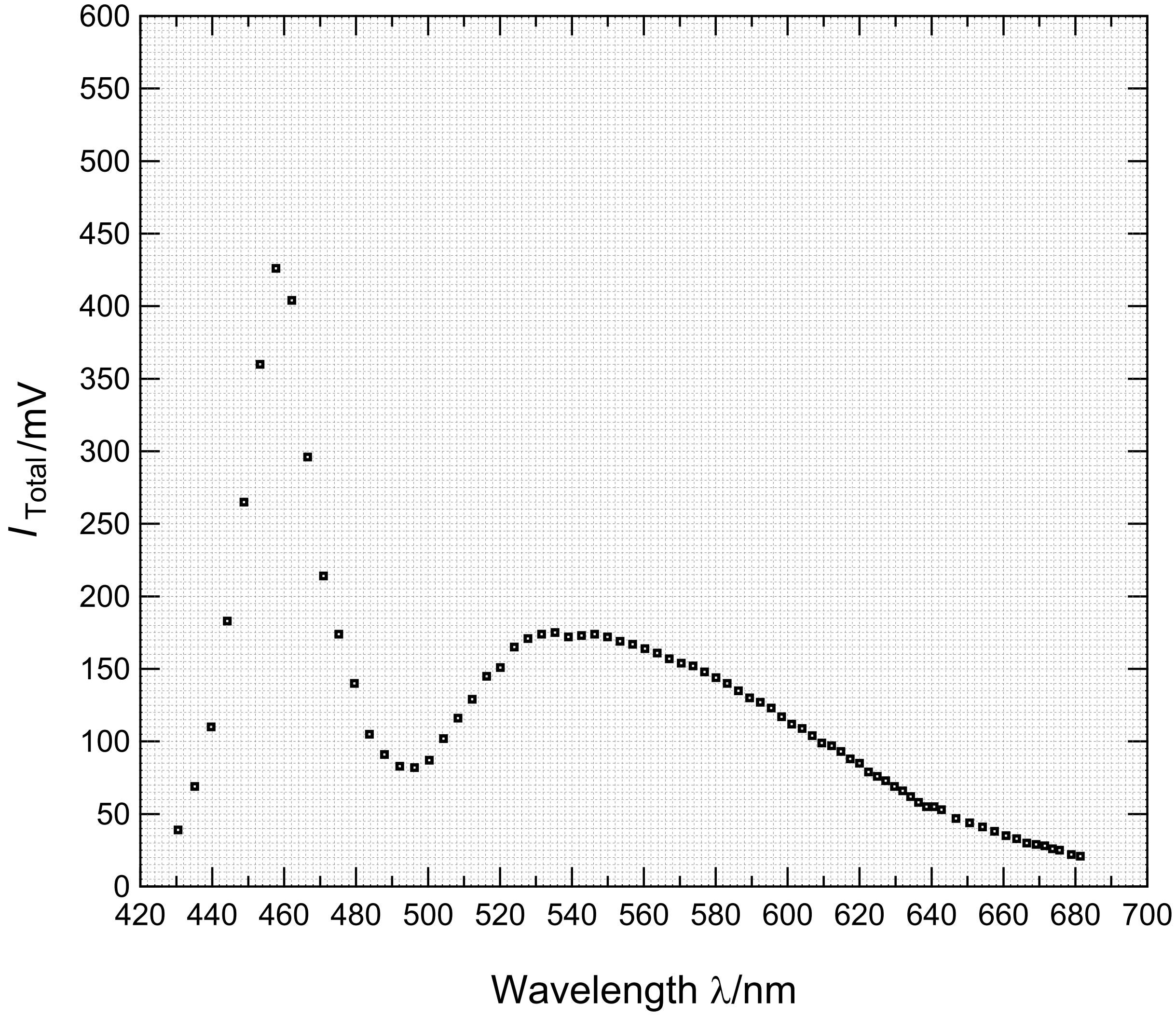
θ_{Stage}/degree	θ/degree	λ/nm	$I_{\perp}$/mV	$I_{\parallel}$/mV	I_{Total}/mV	I_{Norm}
11.5	-12	580.1	127	17	144	0.882
11	-11.5	583.2	114	26	140	0.814
10.5	-11	586.3	97	38	135	0.719
10	-10.5	589.4	80	50	130	0.615
9.5	-10	592.4	67	60	127	0.528
9	-9.5	595.4	54	69	123	0.439
8.5	-9	598.3	41	76	117	0.350
8	-8.5	601.1	31	81	112	0.277
7.5	-8	604.0	22	87	109	0.202
7	-7.5	606.8	15	89	104	0.144
6.5	-7	609.5	8	91	99	0.081
6	-6.5	612.2	6	91	97	0.062
5.5	-6	614.8	4	89	93	0.043
5	-5.5	617.4	3	85	88	0.034
4.5	-5	620.0	4	81	85	0.047
4	-4.5	622.5	5	74	79	0.063
3.5	-4	624.9	7	69	76	0.092
3	-3.5	627.3	10	63	73	0.137
2.5	-3	629.7	12	57	69	0.174

B.1 (2.0 pt)

(Continued)

θ_{Stage}/degree	θ/degree	λ/nm	$I_{\perp}$/mV	$I_{\parallel}$/mV	I_{Total}/mV	I_{Norm}
2	-2.5	632.0	15	51	66	0.227
1.5	-2	634.2	18	44	62	0.290
1	-1.5	636.4	21	37	58	0.362
0.5	-1	638.6	23	32	55	0.418
0	-0.5	640.7	25	30	55	0.455
-0.5	0	642.8	27	26	53	0.509
-1.5	1	646.8	29	18	47	0.617
-2.5	2	650.6	31	13	44	0.705
-3.5	3	654.2	32	9	41	0.780
-4.5	4	657.5	32	6	38	0.842
-5.5	5	660.7	31	4	35	0.886
-6.5	6	663.7	30	3	33	0.909
-7.5	7	666.5	28	2	30	0.933
-8.5	8	669.1	27	2	29	0.931
-9.5	9	671.5	26	2	28	0.929
-10.5	10	673.6	24	2	26	0.923
-11.5	11	675.6	23	2	25	0.920
-13.5	13	678.9	20	2	22	0.909
-15.5	15	681.4	18	3	21	0.857

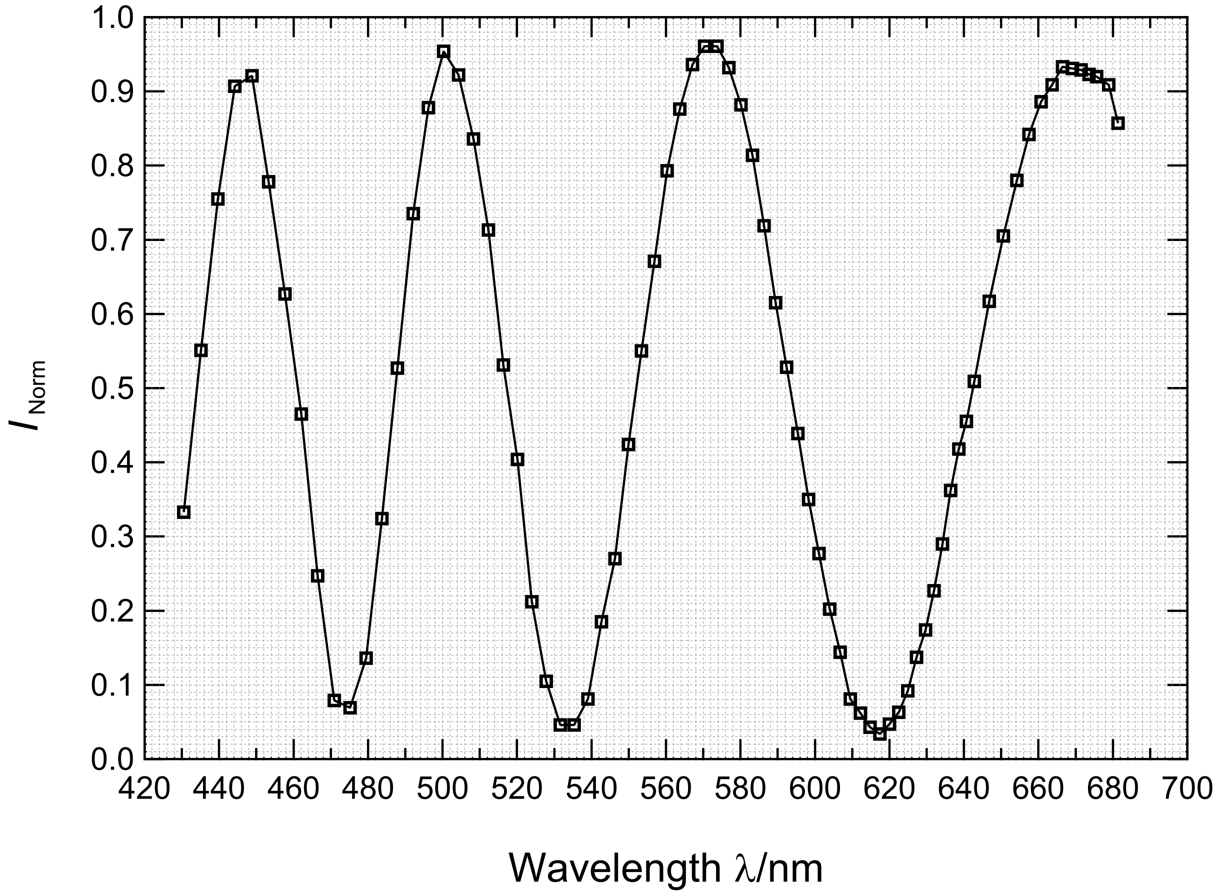
B.2 (1.0 pt)



B.3 (0.2 pt)

$\Delta\lambda_{\mathrm{FWHM}} = 25$ nm $(\Delta\lambda_{\mathrm{FWHM}} \leq 40$ nm$)$

B.4 (1.5 pt)


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
I_{Norm}
420
440
460
480
500
520
540
560
580
600
620
640
660
680
700
Wavelength λ/nm

Part C. Analyses of Measured Results (3.0 points)

C.1 (1.5 pt)

$\lambda = 473\ \mathrm{nm},\ 534\ \mathrm{nm},\ 617\ \mathrm{nm}$

$(455\ \mathrm{nm} < \lambda < 479\ \mathrm{nm},\quad 513\ \mathrm{nm} < \lambda < 539\ \mathrm{nm},\quad 590\ \mathrm{nm} < \lambda < 620\ \mathrm{nm})$

$m = 8,\quad 7,\quad 6$

C.2 (1.5 pt)

$L = 407\ \mu\mathrm{m}\quad (390\ \mu\mathrm{m} < L < 410\ \mu\mathrm{m})$