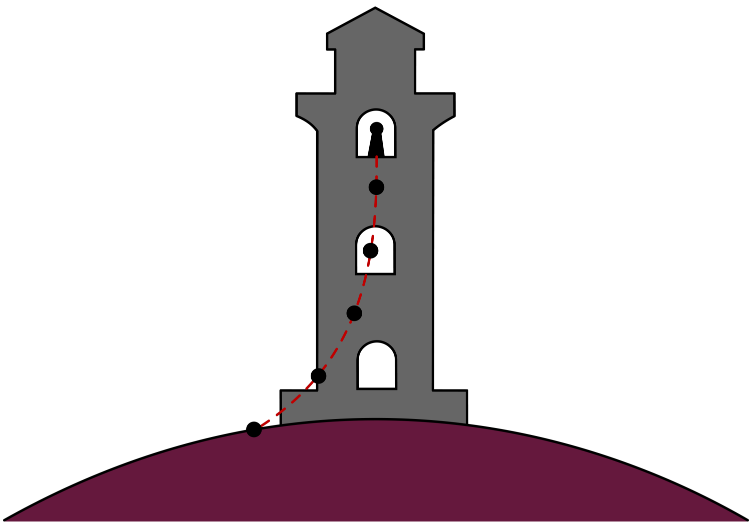
Planet (12 points)

You find yourself on an alien planet with no knowledge of how you got there. The first thing you try to do is to learn more about the planet you're on. You remember how Galileo experimented with falling balls and inspired by this, build a perfectly vertical tower of height $H = 2000\,\text{m}$. Given the tower, you can now start dropping balls from any height h on the tower (measured between the ground and the bottom of the ball immediately before it's released). Due to the limitations of the materials available to you, you can only drop balls of radius $5\,\text{cm} \leq r \leq 50\,\text{cm}$ and densities $0.1\,\text{g/cm}^3 \leq \rho \leq 10\,\text{g/cm}^3$.

Any time you drop a ball, you let it go from rest, and are able to measure the duration t over which it falls, and the horizontal distance s between where the ball lands and the point below where the ball was dropped.

Before you start your experiments, you make the following observations about the planet:

- Based on the movement of the Sun, you find that you're somewhere on the equator of the planet.
- The planet has an atmosphere; the air density is small enough for neglecting the buoyancy force due to it.
- The ground temperature is $T_0 = 20\,^\circ\text{C}$.
- There seems to be a wind blowing along the equator that's uniform throughout the height of the tower; neglect the effect of the tower on the wind velocity.



An artist's exaggerated rendition of the problem.

Description of the simulation software

The command line program simulates the measurements of the fall time t and deflection from the base of the tower s, after providing the height h at which the ball is dropped, its radius r, and density ρ. All values of the input parameters are entered through the keyboard after the corresponding prompts and are validated by pressing the **Enter** key.

In order to get started, use the following authorization key when prompted:

```
Enter Valid Authorization Key: 12345678.888
```

Entering an incorrect value will put the program into test mode; you will need to restart the program.

A typical output of a single simulation cycle of the program looks like:

```
  0 <     h (m)      < 2000 |         h (m):  90
  5 <     r (cm)     < 50   |        r (cm):  13
0.1 < rho (g/cm^3) < 10.0 | rho (g/cm^3):  2
...
t (s) = 3.5, s (m) = 0.1
============================================
  0 <     h (m)      < 2000 |         h (m):_
```

First, you enter the height h in m (the number between 0 and 2000), then the radius of the ball r in cm (the number between 5 and 50) and finally the density of the ball ρ in g/cm^3 (the number between 0.1 and 10). Each input is confirmed with the **Enter** key. The program will then output t in s and s in m.

The program then loops back to the height of the tower query.

Entering a value that is out of range for the experiment will result in an error message,

```
Value Out Of Bounds!
```

and then return you to the incorrectly answered prompt.

The height input h will be rounded to 1 m, r to 1 cm and ρ to 0.01 g/cm^3. (There is no point in trying to input more precise numbers).

The results of the experiment will have random errors associated with them, as to simulate the limited precision one would have in real life. The sizes of the errors can be found by observing the fluctuations in the output.

Any time you need to quit the program, press **Ctrl+C**.

List of constants and useful relations

The gravitational constant $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

Ideal gas constant $R = 8.314$ J mol^{-1} K^{-1},

$0\,^\circ\text{C} = 273.15$ K.

The air drag of a ball of cross-sectional area A and speed v in air of density ρ_a is given by

$$F_d = 0.24 A \rho_a v^2.$$

An adiabatic atmosphere has a density profile given by

$$\rho_a(h) = \rho_{a0}\left(1 - \frac{\gamma - 1}{\gamma}\frac{\mu g h}{R T_0}\right)^{\frac{1}{\gamma-1}} = \rho_{a0}\left(1 - \frac{h}{H_0}\right)^{\frac{1}{\gamma-1}},$$

valid until the top of the atmosphere where $T = 0$ K. Here, γ is the adiabatic coefficient, μ the molar mass of air (i.e. the gas in the atmosphere of the planet), g the free-fall acceleration and h the height from the ground.

Part A. Planetary properties (3.0 points)

A.1	Determine the free-fall acceleration g on the planet by making a suitable set of measurements and sketching an appropriate graph in the space provided. Provide an analysis of the uncertainty in your result.	2.0pt

A.2	Walking away from the tower along the equator, you find that you can see the tower up to a distance of $L = 230$ km away (measured as the distance between you and the top of the tower). What is the radius R of the planet? You may assume that your height is much smaller than the height of the tower.	0.5pt

A.3	Estimate the mass M of the planet. Provide an analysis of the uncertainty in your result. What physical effect contributes the most to the accuracy of your estimate for M ? Tick the appropriate effect the answer sheet.	0.5pt

Part B. Atmospheric properties (6.5 points)

B.1	Determine the wind speed u on the surface of the planet by making a suitable set of measurements and sketching an appropriate graph in the space provided. Provide an analysis of uncertainty in your result.	2.0pt

B.2	Determine the air density ρ_{a0} on the surface of the planet either by collecting additional data or by reusing previous data, and sketching an appropriate graph in the space provided. Provide an analysis of the uncertainty in your result.	1.0pt

B.3	Assuming the atmosphere to be adiabatic with an adiabatic coefficient $\gamma = 1.4$, determine the thickness H_0 of the atmosphere by making a suitable set of measurements and sketching an appropriate graph in the space provided. Provide an analysis of the uncertainty in your result.	3.0pt

B.4	Determine the molar mass μ of the air and the air pressure p_0 at the base of the tower. Provide an analysis of the uncertainty in your result.	0.5pt

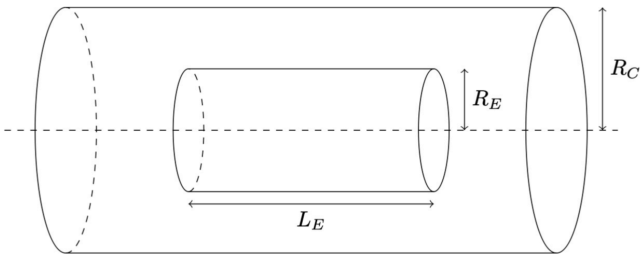
Part C. Duration of a day (2.5 points)

C.1	Determine the duration of a day, T_p, on the planet by making a suitable set of measurements and sketching an appropriate graph in the space provided. Provide an analysis of the uncertainty in your result.	2.5pt

Cylindrical Diode (8.0 pts)

Experimental setup and tasks

A cylindrical vacuum diode consists of two coaxial cylinders. There is an emitter of radius R_E and length L_E, which gives off electrons; these electrons travel through the vacuum to the collector, that has a radius R_C and an effective infinite length. The collector is at a positive potential V, while the emitter is grounded, so electrons are drawn from the emitter to the collector.



The emitter is heated so that there are always excess electrons available to be accelerated through the potential difference toward the collector. The electrons fill the vacuum with a plasma. Because of the properties of a plasma there is a maximum current that can flow through the diode that depends on the potential of the collector and the geometry of the system.

Throughout this experiment you should restrict your measurements to $R_C \geq 5R_E$.

When L_E is sufficiently large compared to R_C it is hypothesized that the maximum current through the diode is

$$I_\infty = G{R_C}^\alpha {L_E}^\beta V^\gamma \tag{1}$$

where $G = G(R_C/R_E)$ is not a constant, but is instead a function of the dimensionless ratio R_C/R_E.

When L_E is comparable to R_C it is necessary to issue a correction to the above expression, and the maximum current through the diodes is given by

$$I_L = I_\infty F(R_C, R_E, L_E, V) \tag{2}$$

where F is a dimensionless function of some or all of R_C, R_E, L_E, and V. Equation (1) is the special case of Equation (2) when $F = 1$

In doing this experiment you have simulated access to any cylinder of radii 0.1 cm to a maximum of 20.0 cm, in steps of 0.1 cm; the cylinder lengths can be between 1.0 cm and 99.0 cm, also in steps of 0.1 cm. There is a simulated power supply that can provide a positive voltage to the collector between 0 and 2000 volts, and an ammeter that can measure the current through the diode.

You are encouraged to read all the tasks through quickly before beginning in order to plan your data collection more efficiently.

Description of the simulation software

The simulation program, named **Exp2**, allows users to perform an unlimited number of measurements of the maximum current I for different sets of input parameters: the collector radius R_C, the emitter radius and length R_E and L_E, and the potential difference between the emitter and the collector V. All values of the input parameters are entered through the keyboard after corresponding prompts and validated by pressing the **Enter** key.

In order to get started, use the following authorization key when prompted:

```
Enter Valid Authorization Key: 12345678.888
```

Entering an incorrect value will put the program into test mode; you will need to restart the program.

A typical interface of a single simulation cycle of the program looks like:

```
0.1 < R_C (cm) < 20.0 | R_C (cm): 18.5
0.1 < R_E (cm) < 20.0 | R_E (cm): 13.2
0.1 < L_E (cm) < 99.0 | L_E (cm): 35.3
1.0 < V_C (V) < 2000.0 | V_C (V): 207

I (A) = 1.04
========================================
0.1 < R_C (cm) < 20.0 | R_C (cm):
```

First you enter the collector radius, then the emitter radius, then the emitter length, each in centimeters, and finally the potential difference, in volts. Each input is confirmed with the **Enter** key.

The program then loops back to the collector radius query.

Entering a value that is out of range for the experiment will result in an error message,

```
Value Out Of Bounds
```

and then return you to the incorrectly answered prompt.

All lengths are only recorded to the nearest millimeter while all voltages are only recorded to the nearest volt; entering in a more precise number does not improve the measurement. However, there is an uncertainty of as much as 0.5 mm in any length, and 0.5 V in any voltage. As such, repeated measurements could give different results for the current.

The ammeter is auto-ranging, so that it shows only three significant figures, and switches between the amp or milliamp scale as appropriate. The uncertainty is $\pm\frac{1}{2}$ of the last displayed digit. Pay attention to whether it is reporting in mA or A.

Exceeding the 40 amp current rating on the ammeter will burn it out. The program will notify you of this, and then automatically fix the ammeter for the next measurement.

Any time you need to quit the program in order to restart, press **Ctrl+C**.

Part A: Finding Exponents (4.5 pts)

Find the exponents in Eq (1), providing an analysis on error bounds on each result:

A.1	Collect a set of data that can be used to find the exponent γ on the variable V. Sketch an appropriate graph in the space provided; for your convenience both linear and log-log graph paper is provided, but you only need to draw one graph. State your value of γ and provide an analysis of the uncertainty in your result.	1.5pt

A.2	Collect a set of data that can be used to find the exponent β on the variable L_E. Sketch an appropriate graph in the space provided; a single graph is sufficient. State your value of β and provide an analysis of the uncertainty in your result.	1.5pt

A.3	Collect a set of data that can be used to find the exponent α on the variable R_C. Sketch an appropriate graph in the space provided; a single graph is sufficient. State your value of α and provide an analysis of the uncertainty in your result.	1.5pt

Part B: Finding the Coefficient G (1.0 pts)

Find the value of the function G when $R_C = 10R_E$:

B.1	Either by collecting additional data or by reusing previous data, determine the value for G when $R_C = 10R_E$ and provide an analysis of uncertainty in your result.	1.0pt

Part C: Finding dimensionless function F (2.5 pts)

Experimentally determine which of R_C, R_E, L_E, and V affect F when L_E is comparable to the size R_C in Equation (2).

C.1	In the list of the variables on the answer sheet, state the direction of the effect; for example, does F increase, decrease, or stay the same if R_C is increased?	0.5pt

C.2	It is observed that when $L_E \approx R_C$ the function F can be approximated as linear in a single variable x, where x is a function of only two from R_C, R_E, L_E, and V. The answer sheet has several possible functional forms for x ; select the one that captures the most significant behavior.	0.5pt

C.3	Assume a linear function of the form $F(x) = A + Bx$ for values of $L_E \approx R_C$, and experimentally determine the parameter B . Restrict to the range $R_C/2 \leq L_E \leq 2R_C$. Sketch an appropriate graph for F in terms of your single appropriate quantity x to approximate F as a linear function. Error analysis is not necessary	1.5pt