General instructions: Experimental Examination (20 points)

July 12, 2016

The experimental examination lasts for 5 hours and is worth a total of 20 points.

Before the exam

- You must not open the envelopes containing the problems before the sound signal indicating the beginning of the competition.
- The beginning and end of the examination will be indicated by a sound signal. There will be announcements every hour indicating the elapsed time, as well as fifteen minutes before the end of the examination (before the final sound signal).

During the exam

- Dedicated answer sheets are provided for writing your answers. Enter the observations into the appropriate tables, boxes or graphs in the corresponding answer sheet (marked A). For every problem, there are extra blank work sheets for carrying out detailed work (marked W). Be sure to always use the work sheets that belong to the problem you are currently working on (check the problem number in the header). If you have written something on any sheet which you do not want to be graded, cross it out. Only use to front side of every page.
- In your answers, try to be as concise as possible: use equations, logical operators and sketches to illustrate your thoughts whenever possible. Avoid the use of long sentences.
- Explicit error calculation is not required unless explicitly asked for. However, you are asked to give an appropriate number of significant digits when stating numbers. Also, you should decide on the appropriate number of data points or measurement repetitions unless specific instructions are given.
- You may often be able to solve later parts of a problem without having solved the previous ones.
- You are not allowed to leave your working place without permission. If you need any assistance (need to refill your drinking water bottle, broken calculator, need to visit a restroom, etc), please draw the attention of a team guide by putting one of the three flags into the holder attached to your cubicle ("Refill my water bottle, please", "I need to go to the toilet, please", or "I need help, please" in all other cases).

At the end of the exam

- At the end of the examination you must stop writing immediately.
- For every problem, sort the corresponding sheets in the following order: cover sheet (C), questions (Q), answer sheets (A), work sheets (W).
- Put all the sheets belonging to one problem into the same envelope. Also put the general instructions (G) into the remaining separate envelope. Make sure your student code is visible in the viewing window of each envelope. Also hand in empty sheets. You are not allowed to take any sheets of paper out of the examination area.

- Put your writing equipment (2 ball point pens, 1 felt tip pen, 1 pencil, 1 pair of scissors, 1 ruler, 2 pairs of earplugs) as well as the provided calculator and your personal calculator (if applicable) back into the transparent zip bag.
- Wait at your table until your envelopes are collected. Once all envelopes are collected your guide will escort you out of the examination area. Take your writing equipment bag with you and hand it in at the exit. Also take your water bottle with you.

Topics

Experiment E-I:	Electrical conductivity in two dimensions	10 marks
Experiment E-II:	Jumping beads - A model for phase transitions and instabilities	10 marks

Experiments E-I and E-II share some of the same equipment. Among others, the same power supply and signal generator are used for both experiments, but with different settings.

Attention: when unpacking the box, do not lift the loudspeaker assembly by the plastic cylinder attached to the membrane.

Material used in both experiments

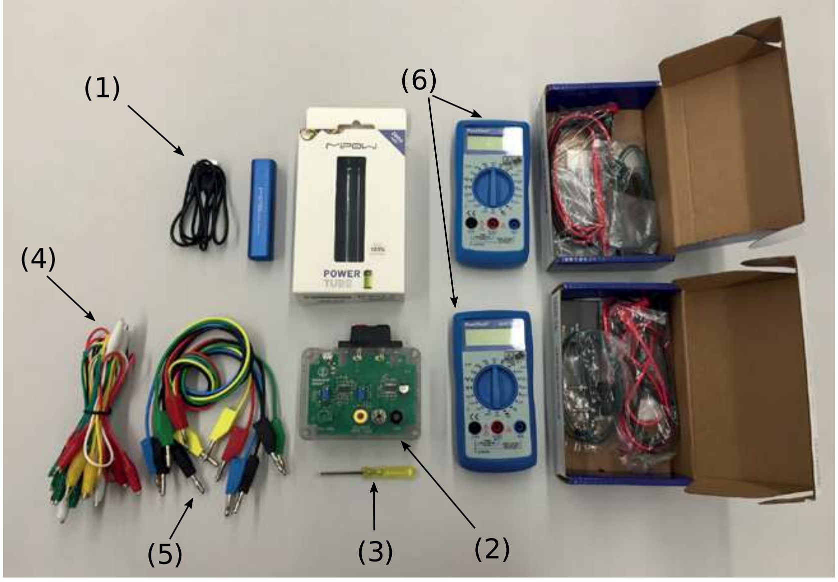


Figure 1: Common material for both experiments.

1. Battery pack with USB cable
2. Adjustable signal generator powered by the battery pack
3. Small screwdriver
4. Ten cables with crocodile clips
5. Six cables with $4\ \mathrm{mm}$ plugs
6. Two digital multimeters

You may also use any of the supplied stationary items to conduct the practical tasks.

Signal generator

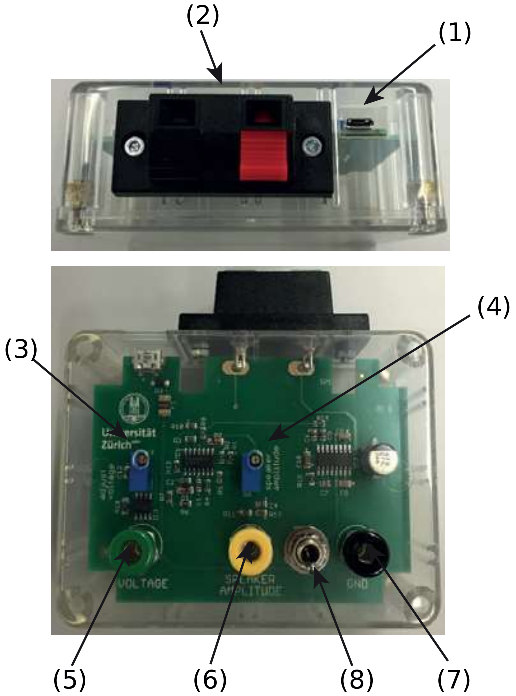


Figure 2.

1. USB connector for powering the signal generator
2. Loudspeaker terminals (only used in E-II)
3. Potentiometer for adjusting the constant voltage (only used in E-I)
4. Potentiometer for adjusting the speaker amplitude (only used in E-II)
5. DC voltage output socket (only used in E-I)
6. Monitor output socket for the loudspeaker drive amplitude (only used in E-II)
7. Common ground socket
8. Switch to turn the loudspeaker terminals and monitor output for the loudspeaker amplitude on / off

To power the signal generator, plug the battery pack using the USB cable to the USB connector of the signal generator (1).

Note that several turns of the potentiometer are required to go from one end of the range to the other. The potentiometers do not have mechanical stops at the end of their range.

Digital multimeters

The digital multimeters can be used for current and voltage measurements. Always connect the two leads to the sockets labeled "$\mathrm{VmA\Omega}$" and "GND" and choose current/voltage and the measurement range by means of the selector.

Electrical conductivity in two dimensions (10 points)

Please read the general instructions in the separate envelope before you start this problem.

Introduction

In the quest to develop next generation devices based on semi-conductor technology like computer chips or solar cells, researchers are looking for materials which exhibit outstanding transport properties, e.g. low electrical resistivity. Measurements of these properties are carried out using samples of finite size, contacts with finite contact resistance and in a special geometry. These effects have to be taken into account in order to extract the true material properties. Moreover, a thin film of the material may behave differently than bulk material.

In this task, we will investigate the measurement of electrical properties. We will use two different definitions:

- **Resistance** R**:** The resistance is the electrical property of a sample or device. It is the quantity which we actually measure on a specific sample with given dimensions.
- **Resistivity** ρ**:** The resistivity is the material property which determines the resistance. It depends on the material itself and on external parameters like the temperature, but it does not depend on the geometry of the sample.

In particular, we will measure the so-called *sheet resistivity*. This is the resistivity divided by the thickness of the very thin sheet.

We will explore the influence of the following parameters on the measurement of the electrical resistance of thin layers of material:

- the measurement circuitry,
- the measurement geometry,
- and the sample dimensions.

A sheet of conductive paper and a metal coated silicon wafer will serve as samples.

List of materials

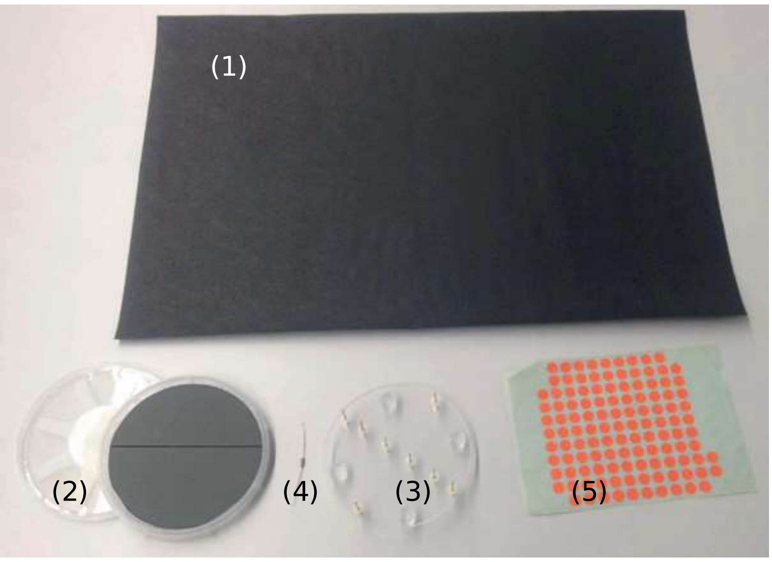


Figure 1: Additional equipment for this experiment.

1. Graphite coated conductive paper
2. A silicon wafer coated with a thin chromium film (stored in a wafer holder)
3. Plexiglas plate with 8 spring-loaded pins
4. An ohmic resistor
5. Color stickers

Important precautions

- The silicon wafer provided can easily be broken if dropped or bent. Do not touch or scratch the shiny metallic surface.

Instructions

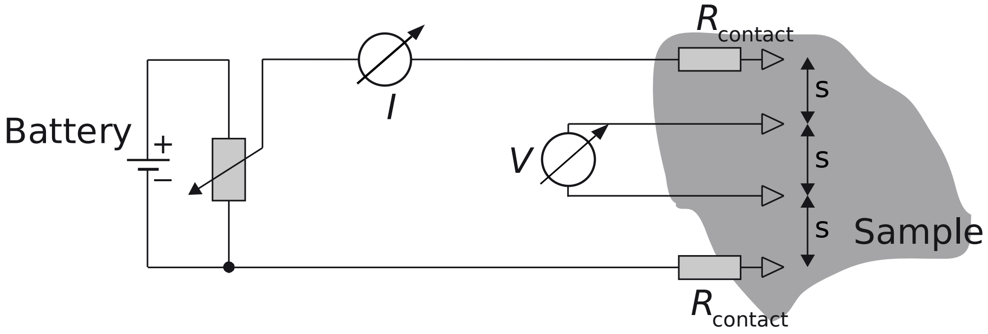
- In the experiment, the signal generator will be used as a DC voltage source. In this mode, the signal generator outputs a constant voltage between the *voltage* socket (5) and the *GND* socket (7). The numbers refer to the photograph shown in the general instructions.
- The voltage (range: 0-$5\ \mathrm{V}$) can be adjusted on the left potentiometer labeled *adjust voltage* (3) using the screwdriver.
- When performing this experiment, make sure that the loudspeaker drive section of the signal generator is turned off using the toggle switch (8). This can be checked by measuring the voltage between the *speaker amplitude* monitor socket (6) and the *GND* socket (7). If the loudspeaker drive section is off, the voltage between these two terminals is zero.

Part A. Four-point-probe (4PP) measurements (1.2 points)

In order to measure the resistivity of a sample precisely, the contacts used for the voltage measurement and the contacts used for current injection should be separated.

This technique is called four-point-probe technique (4PP). The four contacts are arranged into a symmetric geometry that is as simple as possible: The current I flows into the sample through one of the outer contacts (called source), then on all possible paths through the sample and out of the sample through the other contact (drain). In between, the voltage V is measured over a certain path length s on the sample.

Everything becomes quite simple if we have a symmetric setup, i.e. the same distance s between all contacts and the contacts in the center of the sample as shown in following sketch:



The curve I *versus* V represents the $I-V$-characteristics of the sample and allows the resistance of this sample segment to be determined. In the following we will only use the 4PP technique. To start, we will use the linear *equidistant* arrangement of four out of the eight probes (contacts) shown in the photograph.

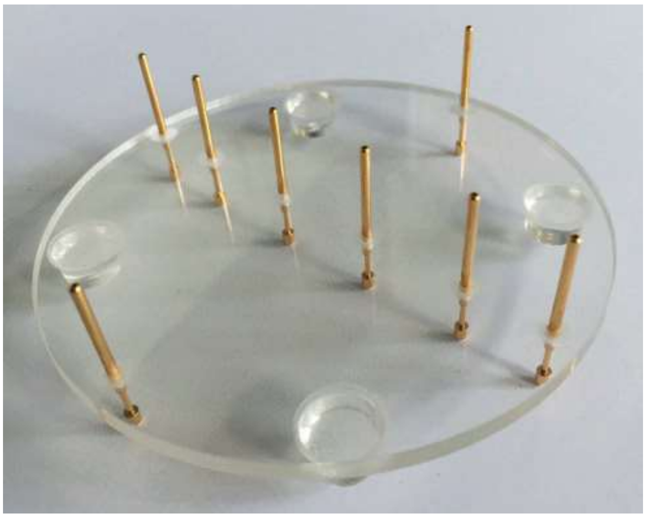


Figure 2: Acrylic glass plate for 4PP measurements, with the four rubber feet and the eight contacts or probes.

For the following measurement, use the whole sheet of conducting paper.

Important hints for all following measurements:

- The long side of the sheet of paper is the reference side. The four probes should be aligned parallel to this side.
- Be careful to use the coated side (black), not the brown back side of the paper! You may mark the correct orientation with color stickers.
- Check that there are no holes or cuts in the paper.
- For these measurements, place the contacts as close to the center of the sample as possible.
- Press the contacts with enough force to ensure good contact for each of them. The plastic feet should just touch the surface.

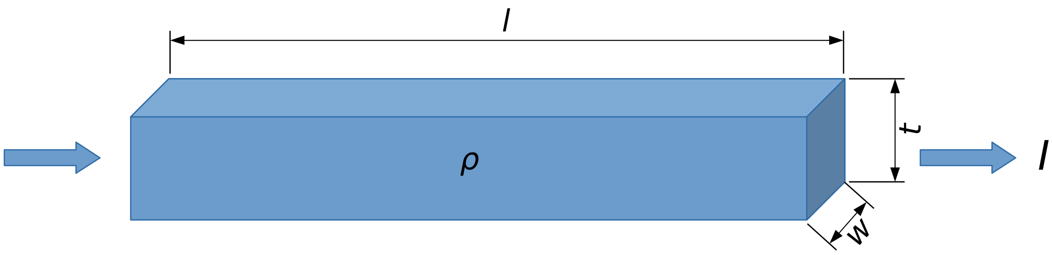
A.1 Four-point-probe (4PP) measurement: Measure the potential drop V over a segment of length s as function of current I passing through this segment. Take in total at least 4 values, make a table and plot the voltage drop V versus the current I in **Graph A.1**. 0.6pt

A.2 Determine the effective electrical resistance $R = \frac{V}{I}$ that you obtained from **Graph A.1**. 0.2pt

A.3 Use **Graph A.1** to determine the uncertainty ΔR on the resistance R for the 4PP measurement. 0.4pt

Part B. Sheet resistivity (0.3 points)

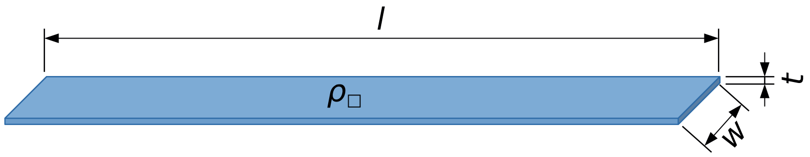
The resistivity ρ represents a material property, by means of which the resistance of a 3D conductor of given dimensions and geometry is calculated. Here we consider a bar of dimensions length l, width w, and thickness t:



The electrical resistance R of the upper, thick conductor is given by:

$$R = R_{3\text{D}} = \rho \cdot \frac{l}{w \cdot t} \tag{1}$$

On the same basis we may define the resistance of the 2D conductor of thickness $t \ll w$ and $t \ll l$



$$R = R_{2\text{D}} = \rho_\square \cdot \frac{l}{w}, \tag{2}$$

using the *sheet resistivity* $\rho_\square \equiv \rho/t$ ("rho box"). Its unit is given in Ohms: $[\rho_\square] = 1\,\Omega$.

Important: Eq. 2 is only valid for a homogeneous current density and constant potential in the cross-sectional plane of the conductor. In the case of point-like contacts on the surface this does not hold. Instead one can show that the sheet resistivity is related to the resistance in that case by

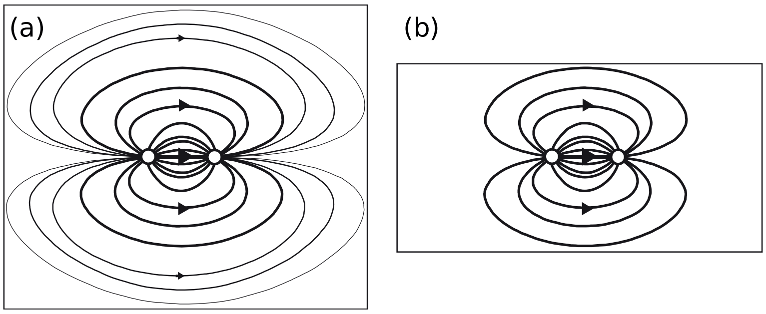
$$\rho_\square = \frac{\pi}{\ln(2)} \cdot R \tag{3}$$

for $l, w \gg t$.

B.1	Calculate the sheet resistivity $\rho_\square$ of the paper from the 4PP measurement in part A. We will call this particular value ρ_∞ (and the measured resistance from part A R_∞) because the sample dimensions of the whole sheet are much larger than the spacing of the contacts s: $l, w \gg s$.	0.3pt

Part C. Measurements for different sample dimensions (3.2 points)

Up to now, the finite sample dimensions w and l were not taken into account. If the sample becomes smaller, it can carry less current if the voltage is kept constant: If we apply a voltage between the two point contacts (white circles), current will flow on all possible, non-crossing paths through the sample as visualized by the lines: the longer the line, the smaller the current as indicated by the line thickness. For a small sample (b) and the same applied voltage, the total current decreases because there are less possible pathways. Thus, the measured resistance will increase:



The (sheet) resistivity will not change as function of sample size. Thus, in order to convert the measured resistance into a resistivity using Eq. 3, we need to introduce a correction factor $f(w/s)$:

$$\rho_\square = \frac{\pi}{\ln(2)} \cdot \frac{R(w/s)}{f(w/s)}. \tag{4}$$

For a sample of length $l \gg s$ the factor f only depends on the ratio w/s and is larger than 1: $f(w/s) \geq 1$. For the sake of simplicity we will focus on the dependence on the width w and only ensure that the sample is long enough for our measurements. We assume that the value approaches the correct result $\rho_\square$ for large dimensions:

$$R(w/s) = R_\infty \cdot f(w/s) \quad \text{with} \quad f(w/s \to \infty) \to 1.0. \tag{5}$$

C.1	Using the 4PP-method, measure the resistance $R(w,s)$ for 4 values w/s within the range 0.3 to 5.0 and record your results in **Table C.1**. Ensure that the sample length is larger than five times the probe spacing: $l > 5s$ and that the length l of the samples is always taken along the same (long) side of the sheet of paper. For each value of w/s measure the voltage for 4 different current values and calculate the average resistance $R(w/s)$ out of the 4 measurements. Enter your results in **Table C.1**.	3.0pt

C.2	Compute $f(w/s)$ for each of these measurements.	0.2pt

Part D. Geometrical correction factor: scaling law (1.9 points)

You have seen in part C that the measured resistivity scales with the ratio of width to probe distance w/s. Starting from the data acquired in part C we choose the following generic function to describe the data in the range of the measurements:

$$\text{Generic fit function:} \quad f(w/s) = 1.0 + a \cdot \left(\frac{w}{s}\right)^b \tag{6}$$

Note that for very large w/s, $f(w/s)$ must be 1.0.

D.1	In order to fit a model curve using Eq. 6 and the data $f(w/s)$, taken in part C, choose the most appropriate graph paper (linear **Graph D.1a**, semi-logarithmic **Graph D.1b**, or double-logarithmic **Graph D1.c**) to plot the data.	1.0pt

D.2	Deduce the parameters a and b from your fit.	0.9pt

Part E. The silicon wafer and the van der Pauw-method (3.4 points)

In the semi-conductor industry, knowledge of the electrical (sheet) resistance of semi-conductors and thin metal layers is very important because it determines the properties of devices. In the following you will work with the silicon wafer. The semi-conducting wafer is coated with a very thin layer of chromium metal (on the shiny side).

Open the wafer container (rotate in the sense of the arrow RELEASE) and take the wafer out. Be careful not to drop or to break it nor to scratch or touch the shiny surface. For the measurements place it on the table with the shiny side point up towards you.

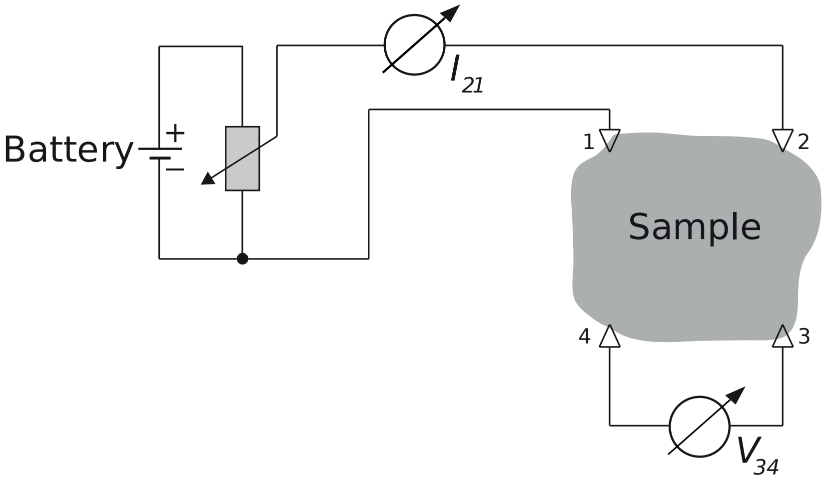
E.1	Use the same 4PP setup as previously to measure the voltage V as function of current I. Write down the reference number of your wafer in the Answer Sheet. You find this number on the plastic wafer holder.	0.4pt

E.2	Plot the data in **Graph E.2** and determine the resistance $R_{\rm 4PP}$.	0.4pt

E.3	In order to determine the correction for a circular sample like the wafer, we will approximate the effective width w of the sample by the diameter $D = 100\,\rm mm$ of the wafer. Under this assumption calculate the ratio w/s. Use the fit function in Eqn. 6 and your parameters a and b to determine the correction factor $f(w/s)$ for the wafer measurement.	0.2pt

E.4	Calculate the sheet resistivity $\rho_\square$ of the chromium layer using Eq. 4.	0.1pt

In order to measure the sheet resistivity precisely without need for geometrical corrections, Philips engineer L.J. van der Pauw developed a simple measurement scheme: The four probes are mounted at the circumference of a sample of arbitrary shape as shown in the figure (numbered 1 through 4). The current flows through two adjacent probes, e.g. probes 1 and 2, and the voltage is measured between probes 3 and 4. This yields a resistance value $R_{I,V} = R_{21,34}$.



For symmetry reasons $R_{21,34} = R_{34,21}$ and $R_{14,23} = R_{23,14}$. Van der Pauw showed that for an arbitrary but simply connected shape (no holes) of the sample and point-like contacts the following equation holds:

$$e^{-\pi R_{21,34}/\rho_\square} + e^{-\pi R_{14,23}/\rho_\square} \equiv 1. \tag{7}$$

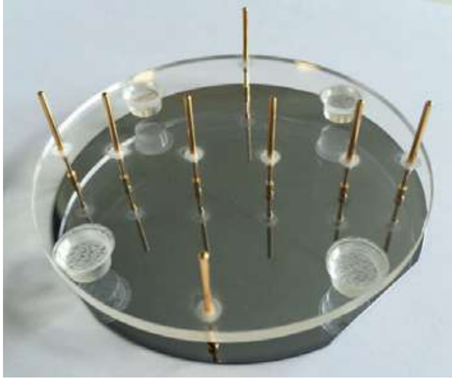


Figure 3: 4PP device on the metal coated silicon wafer. Note the cut on the right-hand-side of the circular wafer. This cut is called flat.

Connect the four spring contacts such that the measurement probes form a square. Connect two adjacent contacts to the current source with the amperemeter, and connect the two remaining spring contacts with the voltmeter. Rotate the square until one of its edges is parallel to the flat of the wafer.

E.5	Sketch the orientation of the current carrying contacts and the orientation of the flat of the wafer. Measure the voltage V for at least in total 6 different values of current I, roughly equally spaced. Enter the results into **Table E.5**.	0.6pt

E.6	Repeat the procedure arranging the current carrying contacts perpendicular to those used in the first step. Enter the results into **Table E.6**.	0.6pt

E.7	Plot all the data together in a single graph **Graph E.7** using different colors and/or symbols. Determine the mean value $\langle R \rangle$ from the two curves.	0.5pt

E.8	Replacing all resistances $R_{kl,mn}$ by $\langle R \rangle$, solve Eqn. 7 for $\rho_\square$ and calculate the sheet resistivity $\rho_\square$ of the chromium layer.	0.4pt

E.9	Compare the result of the measurement taken with the linear arrangement (**E.4**) and the result of the van der Pauw method (**E.8**). Give the difference of the two measurements as relative error in percent.	0.1pt

E.10	The chromium (Cr) layers have a nominal thickness of $8\ \mathrm{nm}$. Use this value and the final results of the van der Pauw method to calculate the resistivity of Cr using Eqns. 1 and 2.	0.1pt

Jumping beads - A model for phase transitions and instabilities (10 points)

Please read the general instructions in the separate envelope before you start this problem.

Introduction

Phase transitions are well known from every day life, e.g. water takes different states like solid, liquid and gaseous. These different states are separated by phase transitions, where the collective behaviour of the molecules in the material changes. Such a phase transition is always associated with a transition temperature, where the state changes, i.e. the freezing and boiling temperatures of water in the above examples.

Phase transitions are however even more wide-spread and also occur in other systems, such as magnets or superconductors, where below a transition temperature the macroscopic state changes from a paramagnet to a ferromagnet and a normal conductor to a superconductor, respectively.

All of these transitions can be described in a common framework when introducing a so-called order-parameter. For instance, in magnetism the order parameter is associated with the alignment of the magnetic moments of the atoms with a macroscopic magnetisation.

In the so-called continuous phase transitions, the order parameter will always be zero above the critical temperature and then grow continuously below it, as shown in the schematic for a magnet in figure 1 below. The transition temperature of a continuous phase transition is called the critical temperature. The figure also contains a schematic representation of the microscopic order or disorder in the case of a magnet, where the individual magnetic moments align in the ferromagnetic state to give rise to a macroscopic magnetization, whereas they are randomly oriented in the paramagnetic phase yielding a macroscopic magnetization of zero.

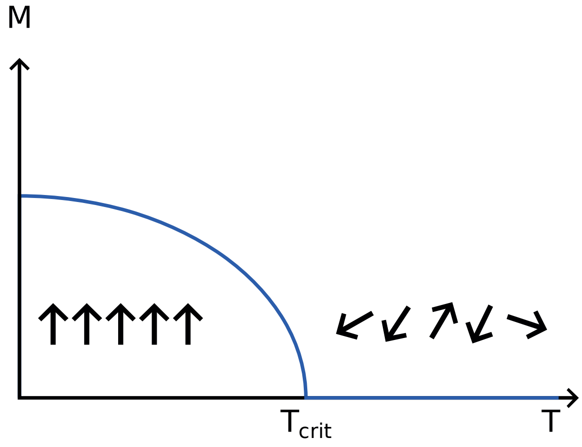


Figure 1: Schematic representation of the temperature dependence of an order parameter M at a phase transition. Below the critical temperature T_{crit}, the order parameter grows and is non-zero, whereas it is equal to zero at temperatures above T_{crit}.

For continuous phase transitions, one generally finds that the order parameter close to a transition follows a power-law, e.g. in magnetism the magnetization M below the critical temperature, T_{crit}, is given by:

$$M \begin{cases} \sim (T_{\mathrm{crit}} - T)^b, & T < T_{\mathrm{crit}} \\ = 0, & T > T_{\mathrm{crit}} \end{cases} \tag{1}$$

where T is temperature. What is even more stunning is that this behaviour is universal: the exponent of this power-law is the same for many different kinds of phase transition.

Task

We will study a simple example where some of the features of continuous phase transitions can be investigated, such as how an instability leads to the collective behaviour of the particles and thus to the phase transition as well as how the macroscopic change depends on an excitation of the particles.

In common phase transitions this excitation is usually driven by temperature. In our example, the excitation consists of the kinetic energy of the particles accelerated by the loudspeaker. The macroscopic change corresponding to the phase transition that we study here consists of the sorting of beads into one half of a cylinder, which is separated by a small wall.

Increasing the amplitude from where particles have sorted into one half of the cylinder, you will find that eventually the particles distribute equally between the two halves. This corresponds to having heated past the critical temperature.

Your objective is to determine the critical exponent for the model phase transition studied here.

List of material

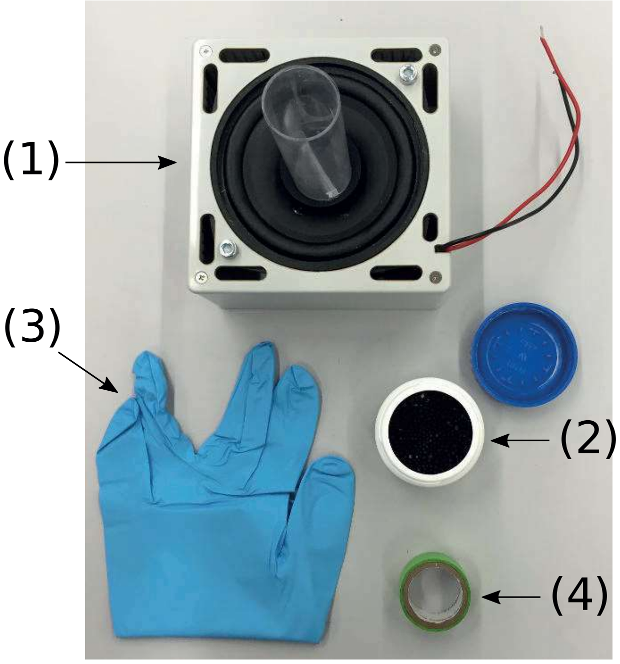


Figure 2: Additional equipment for this experiment.

1. Loudspeaker assembly with plastic cylinder mounted on top
2. About 100 poppy seeds (in a plastic container)
3. A glove
4. Sticky tape

Important precautions

- Do not apply an excessive lateral force to the plastic cylinder mounted on the loudspeaker. Note that no replacements will be provided in case of torn loudspeaker membranes or torn off plastic cylinder.
- Turn off the loudspeaker assembly whenever not in use, in order to avoid unnecessary drain of the battery.
- In this experiment, a $4\ \mathrm{Hz}$ saw-tooth signal is output on the loudspeaker terminals located at the side of the signal generator.
- The amplitude of the saw-tooth signal can be adjusted using the right potentiometer labeled *speaker amplitude* (4). A DC voltage proportional to the signal amplitude is output on the *speaker amplitude* monitor socket (6) (with respect to the *GND* socket (7)). The numbers refer to the photograph (Figure 2) shown in the general instructions.
- The speaker membrane is delicate. Make sure that you do not apply unnecessary pressure on it by any means either vertically or laterally.

Part A. Critical excitation amplitude (3.3 points)

Before you start the actual tasks of this problem, wire up the loudspeaker to the terminals on the side of the signal generator (make sure you use the correct polarity). Put some (e.g. 50) poppy seeds into the cylinder mounted on the loudspeaker and use a piece cut from the glove provided to close the cylinder at the top in order to keep the poppy seeds in the cylinder. Switch on the excitation using the toggle switch and adjust the amplitude by turning the right potentiometer labeled *speaker amplitude* (4) by means of the screwdriver provided. Observe the sorting of the beads by testing different amplitudes.

The first task is to determine the critical excitation amplitude of this transition. In order to do this, you have to determine the number of beads N_1 and N_2 in the two compartments (choosing the compartment labels such that $N_1 \leq N_2$) as a function of the displayed amplitude A_D, which is the voltage measured at the *speaker amplitude* socket (6). This voltage is proportional to the amplitude of the saw-tooth waveform driving the loudspeaker. Make at least 5 measurements per voltage.

Hint:

- In order to always have a motion in the particles you study, only investigate amplitudes corresponding to *speaker amplitude* voltages exceeding $0.7\ \mathrm{V}$. Start with watching the behaviour of the system just by varying the voltage slowly without any counting of he beads. It can be that some of the beads stick to the ground due to electrostatic reasons. Don't count these beads.

A.1	Record your measurements of the number of particles N_1 and N_2 in each half of the container for various amplitudes A_D in **Table A.1**.	1.2pt

A.2	Calculate the standard deviation of your measurements of N_1 and N_2 and list your results in **Table A.1**. Plot N_1 and N_2 as a function of the displayed amplitude A_D in **Graph A.2**, including their uncertainties.	1.1pt

A.3	Based on your graph, determine the critical displayed amplitude $A_{D,\mathrm{crit}}$ at which $N_1 = N_2$, after waiting until a stationary state is reached.	1pt

Part B. Calibration (3.2 points)

The displayed amplitude A_D, corresponds to a voltage applied to the loudspeaker. However, the physically interesting quantity is the maximum displacement A of the oscillation of the loudspeaker, since this relates to how strongly the beads are excited. Therefore, you need to calibrate the displayed amplitude. For this purpose, you can use any of the provided material and tools.

B.1	Sketch the setup you use to measure the excitation amplitude, i.e. the maximum travel distance A (in mm) of the loudspeaker in one period of oscillation.	0.5pt

B.2	Determine the amplitude A in mm for a suitable number of points, i.e. record the amplitude A as a function of displayed amplitude A_D in **Table B.2** and indicate the uncertainties of your measurements.	0.8pt

B.3	Plot your data in **Graph B.3**, including the uncertainties.	1.0pt

B.4	Determine the parameters of the resulting curve, using an appropriate fit to determine the calibration function $A(A_D)$.	0.8pt

B.5	Determine the critical excitation amplitude A_{crit} of the poppy seeds.	0.1pt

Part C. Critical exponent (3.5 points)

In our system, the temperature corresponds to the input kinetic energy of the excitation. This energy is proportional to the speed squared of the loudspeaker, i.e. to $v^2 = A^2 f^2$, where f is the frequency of the oscillation. We will now test this dependence and determine the exponent b of the power-law governing the behavior of the order parameter (see Eq. 1).

C.1	The imbalance $\left\|\frac{N_1-N_2}{N_1+N_2}\right\|$ is a good candidate for an order parameter for our system in that it is zero above the critical amplitude and equal to 1 at low excitation. Determine this order parameter as a function of the amplitude A. Record your results in the **Table C.1**.	1.1pt

C.2	Plot the imbalance $\left\|\frac{N_1-N_2}{N_1+N_2}\right\|$ as a function of $\left\|A_{\text{crit}}^2 - A^2\right\|$, in **Graph C.2**, where both axes have logarithmic scales (double-logarithmic plot). You can use the **Table C.1** for your calculations. The points on the plot may seem not to obey a linear relation, but a linear regression should be made nevertheless, to match the critical exponent formula.	1pt

C.3	Determine the exponent b and estimate the error.	1.4pt