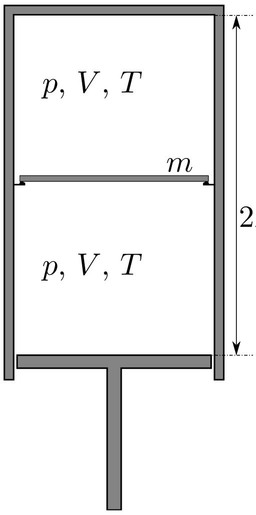
T1: A leak

A hollow insulated cylinder of height $2H$ and volume $2V$ is closed from below by an insulating piston. The cylinder is divided into two initially identical chambers by an insulating diaphragm of mass m. The diaphragm rests on a circular ledge and a gasket between them provides tight contact. Both chambers are filled with gaseous helium at pressure p and temperature T. A force is applied to the piston, so that it moves upwards slowly.



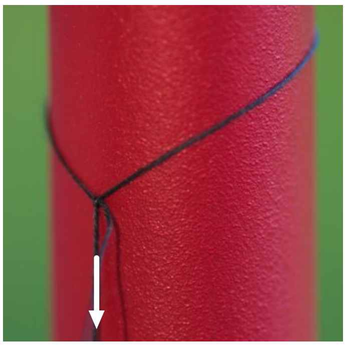
a. Find the volume of the lower chamber V_0 when the gas starts to leak between the chambers.

b. Find the temperature T_1 in the upper chamber when the piston touches the diaphragm.

c. Find the temperature T_2 in the lower chamber immediately before the piston touches the diaphragm.

T2: Thread around a cylinder

One end of a thread is tied into a loop of length $L > 2\pi R$, and a cylinder of radius R is put through the loop. The coefficient of friction between the thread and the cylinder is μ. The free end of the thread is being pulled parallel to the axis of the cylinder (as shown by arrow in the photo) while keeping the cylinder at rest. If the length of the loop is longer than a critical value, $L > L_0$, the loop can slide along the cylinder without changing its shape, otherwise the friction "locks" it into a place and increasing the pulling force would eventually just break the thread. Find this critical value L_0. The weight of the thread is to be neglected; the thread will not twist when being pulled.



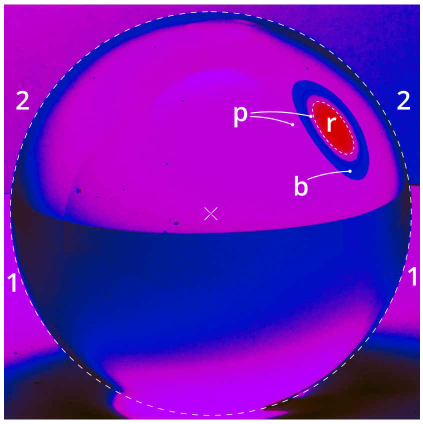
It might be useful to know that

$$2\int \sqrt{1+x^2}\,\mathrm{d}x = x\sqrt{1+x^2} + \operatorname{arcsinh} x\,,$$

where $\operatorname{arcsinh} x \equiv \ln\left(x+\sqrt{1+x^2}\right)$.

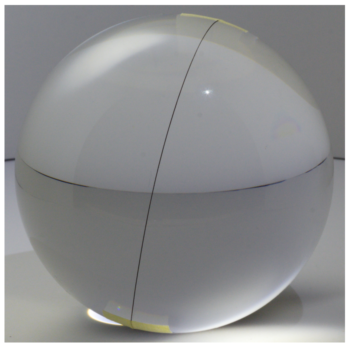
T3: Glass ball

The first photo here is taken with a digital camera and shows a glass ball, backlit with diffuse dichromatic light which has only two narrow spectral lines (red 630 nm and violet 400 nm). This diffuse light comes from the white floor (marked with '1' in the figure) and the white walls (marked with '2'), both illuminated with violet and red LED lamps. The sensor of the camera has only red, blue and green sensors so that violet light appears in the photo as blue. The photo is taken from a distance much greater than the radius of the ball. On the back side of the ball, a very narrow opaque thread is glued to the glass surface, forming an arc of a great circle on the ball. In the photo, the thread is obstructed by the ball and cannot be seen directly. However, hugely deformed images of a very short segment of the thread are seen as blue (marked with 'b') and red ('r') ellipses. The letter 'p' indicates purple-coloured areas in the photo.



In the first photo, the centre of the ball is marked with a cross, and the perimeter of the ball is traced with a dashed line. You can find a larger version of the first photo on a separate sheet. You may take distance measurements there. In the larger photo, the boundary between the red and purple regions is also traced with a dashed line.

The second photo is taken while illuminating with a white LED with the ball turned so that the thread can be seen directly.



a. Explain qualitatively using a ray diagram why a segment of the thread is seen as a closed loop in the first photo.

b. Determine the coefficient of refraction n_R for the red light.

c. Determine the difference of the coefficients of refraction $\Delta n \equiv n_V - n_R$ for the red and violet light (with n_V denoting the coefficient of refraction for the violet light).