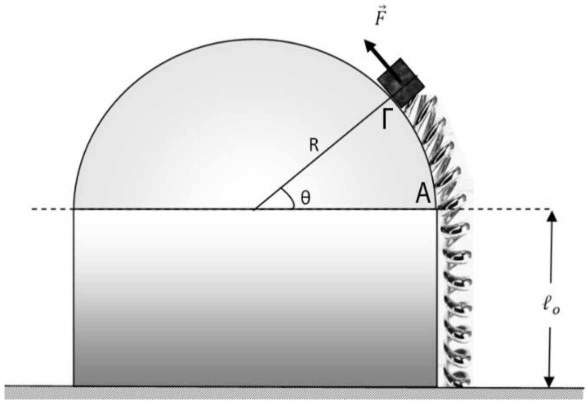
24 June 2023, Durrës
ALBANIA

Problems

1. *Springs*



A. (10 points) A conservative variable force F is slowly pulling a body of weight W as illustrated in the figure, along a smooth semi-sphere with a radius R. The force acts tangential to the semi-sphere. When the body is in the position A the spring has its natural length.
Calculate the work done by F, to move the body from A to Γ.

B. (15 points) For a particular horizontal spring, the intensity of the elastic force, depends on the deformation x as follows: $F(x) = \alpha \cdot x + \beta$, where $\alpha = 50$ N/m and $\beta = 10$ N.

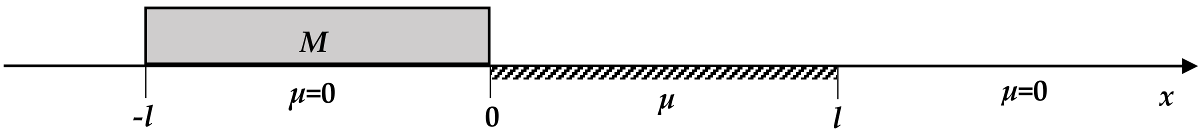
B1. (5 points) Calculate the potential energy function $U(x)$ for this spring. Assume that $U(0) = 0$.

B2. (10 points) An object of mass 2 kg is attached to the end of this spring and it's elongated with 1.5 m on a smooth horizontal surface and then released. Determine the speed of the object when the elongation is 1.0 m for the first time.

2. *Plank*

A plank of mass M and length l lies on a smooth horizontal surface, and it can move across a rough area (*i.e.* along the x axis), characterized by its length l and the sliding friction coefficient μ, the same as the static one. The initial position of the plank is that depicted in the figure below.

A. (6.3 points) The plank is launched with the unknown initial, horizontal speed v_0, towards the rough area. Derive the minimum initial speed of the plank for which:



A1. (5 points) it fully enters the rough area.

A2. (1.3 points) it completely surpasses the rough area.

B. (3.7 points) The plank starts from rest as illustrated in the figure above but is pulled to the right by a constant horizontal force F_0, permanently acting on it. The purpose is to pull the plank on the rough area.

B1. (2.5 points) Determine the minimum value of the force for the plank to completely enter the rough area.

B2. (1.2 points) Derive the maximum value of the plank's speed during its motion analyzed at B1, for the minimum value found for F_0.

C. (15 points) The plank starts from rest as illustrated in the figure above but is pulled to the right by a constant horizontal force F_0, permanently acting on it. The purpose is to make the plank surpass the rough area for a minimum value of F_0.

C1. (3.7 points) Make a graph representation of the net force acting on the plank versus the coordinate x of its front end for $x \in [0, 3l]$.

C2. (5 points) Determine the minimum value of the force for the plank to completely exit the rough area. It is known that the minimum value of the plank kinetic energy is very small, *i.e.* a very small fraction ε of the maximum value of the kinetic energy the plank had until it reached the minimal value. From the mathematical point of view, the fact that $\varepsilon \ll 1$ means that its algebraic powers higher than one can be neglected. The value of ε is known.

C3. (6.3 points) Derive the maximum value of the plank's speed during its motion on the interval $x \in [0, 2l]$. Plot the graph of the plank' speed as a function of x, for $x \in [0, 3l]$.

Note: *If useful, you can use the Bernoulli's approximation* $(1 + x)^n \cong 1 + nx$, *if* $|x| \ll 1$.

3. ***Tennis ball***

A. Warming up. (15 points)

During warm up, a tennis ball (A) is dropped from Novak Djokovic's pocket at a height H. At the same time, a grasshopper (B) just underneath Novak's pocket, jumps from the ground, vertically towards the ball with initial velocity v_0. When they collide, the ball has twice the speed of the grasshopper. The collision occurs at height h.

A1. (1 point) Write down the equation of motion for the ball, y_A as a function of time, in terms of H, v_0 and gravitational acceleration g.

A2. (1 point) Write down the equation of motion for the grasshopper, y_B as a function of time, in terms of v_0 and g.

A3. (3 points) Derive the expression for the velocity of the ball $v_A(t)$ in terms of H, g and $y_A(t)$.

A4. (1 point) Derive the expression for the velocity of the grasshopper $v_B(t)$ in terms of v_0, g and $y_B(t)$.

A5. (3 points) Derive the expression for the initial velocity v_0 of the grasshopper in terms of h, H and g.

A6. (3 points) Derive the expression for the moment of the collision t_c in terms of v_0 and g.

A7. (3 points) Calculate the numerical value of the ratio h/H.

B. Match (10 points)

During match, at serve, Novak aims to hit tennis ball horizontally.

B1. (1 point) Write down the expression for the equation of motion of the ball in the horizontal direction $x(t)$.

B2. (1 point) Write down the expression for the equation of motion of the ball in the vertical direction $y(t)$.

B3. (3 points) Calculate the numerical value of the minimal initial velocity v_i required for the ball to pass just above the 0.9 m high net, 15 m in front of Novak, if the ball is launched (horizontally) from a height of 2.5 m.

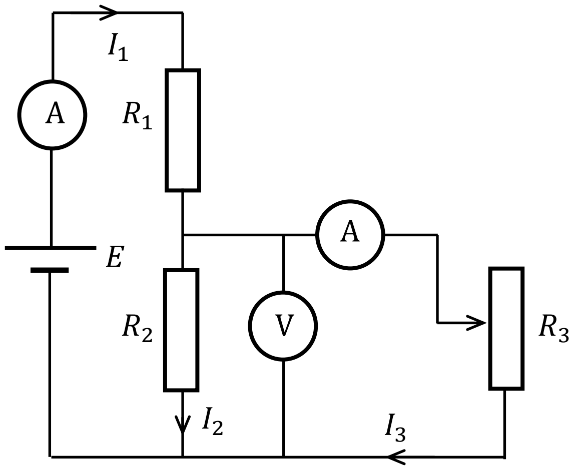
B4. (4 points) Where will the ball land in the case given under B3?

B5. (1 point) How long will the ball be in the air before it lands in the case given under B3?

The gravitational acceleration is $g = 9.8\ m/s^2$.

4. ***Electric circuit***

The circuit diagram shown in the figure contains a battery with voltage E, two resistors of fixed value R_1 and R_2, one resistor R_3, the value of which can be changed, one voltmeter and two ammeters. A series of measurements were made at different values of the resistor R_3, which are given in the table below. The measured electric currents are given in the figure (I_1, I_2, I_3). From the data presented, calculate (all additionally calculated data must be present in the empty columns of the table):



N	I_1/mA	I_3 /mA	U/V					
1	5.73	5.46	0.545					
2	5.31	4.62	1.38					
3	4.88	3.75	2.25					
4	4.58	3.16	2.84					
5	4.36	2.73	3.27					
6	4.20	2.40	3.60					
7	4.07	2.14	3.86					
8	3.97	1.94	4.07					
9	3.75	1.50	4.50					

A. (2 points) The value of the resistance R_2.

B. (4 points) Using suitable variables, represent some of the measurements on such a graph that the values of voltage E and resistance R_1 can easily be obtained. Draw the dependence of the two variables on the graph paper provided.

C. (2 points) Using the obtained graph from B, calculate the values of the voltage E and the resistance R_1.

D. (5 points) Let's define the efficiency η of the circuit as the ratio of the electrical power dissipated in the resistor R_3 to the power dissipated in the entire circuit. Draw the dependence of the efficiency η on the value of the resistance R_3 in the second graph paper provided.

E. (2 points) Using the obtained graph from D, determine the maximal value of the efficiency η, as well as the value of R_3 for which it is obtained.

F. (5 points) If $R_1 = R_2 = R$, calculate the theoretical maximum for the efficiency η.

G. (4 points) Determine the value of R_3 (in terms of R) for which this maximum η is obtained.

H. (1 point) Compare the theoretical and experimental results (the values of η and R_3).