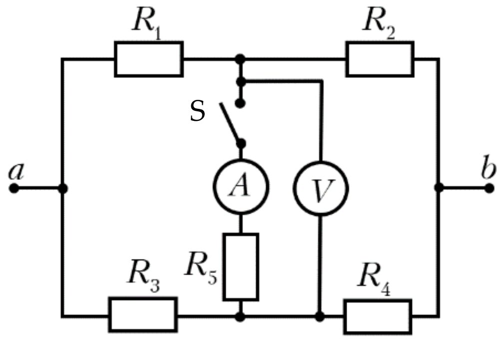
Problem 1: Resistor circuit



A constant voltage U_0 is applied to the *ab* ends of the circuit in figure. The resistors have resistances $R_1=R_2=R_5=R$, $R_3=2R$, $R_4=4R$. The voltmeter and the ammeter are considered ideal.

A. (**9 points**) With the **switch S open** determine:

A1. (4 points) The equivalent resistance, R_{ab}, between the *ab* ends of the circuit.

A2. (**3 points**) The values measured by the ammeter and the voltmeter.

A3. (**2 points**) The power dissipated on the resistor R_4.

B. (**13 points**) With the **switch S closed** determine:

B1. (**9 points**) The equivalent resistance, R_{ab}, between the *ab* ends of the circuit.

B2. (**2 points**) The values measured by the ammeter and the voltmeter.

B3. (**2 points**) The power dissipated on the resistor R_4.

C. (**3 points**) The switch S is closed and opened periodically, so that in a time period *T*, *2T/3* of the time is closed and *T/3* of the time is opened. Determine the average power dissipated on the resistor R_4.

Assume that all current values are reached instantaneously.

Problem 2: The descent of a skier

A skier descent a steep hill. The angle between the inclined surface of the hill and a horizontal plane is $\alpha = 30{,}0°$. The acceleration of gravity is $g = 9{,}81\ \mathrm{m/s^2}$. The mass of the skier is $m = 80{,}0$ kg. The coefficient of friction μ of the skies in the snow is unknown. The drag force F_D with which the air acts on the skier, is $F_D = \frac{1}{2}C\rho A v^2$, where C is an unknown number, the so called drag coefficient, depending on the shape of the skier, $\rho = 1{,}28\ \mathrm{kg/m^3}$ is the density of the air, $A = 0{,}600\ \mathrm{m^2}$ is the projected frontal area (cross section) of the skier perpendicular to his velocity, and v is his speed relative to the air. The air does not move (there is no wind). The skier's descent is recorded. Analyzing the video, some of his positions x_i (measured on the slope from the starting point of descent) in different moments t_i after the start of his descent are measured. The data are shown on the table below.

i	t_i/s	x_i/m						
1	2,00	7,21						
2	2,50	11,22						
3	3,00	16,08						
4	3,50	21,76						
5	4,00	28,24						
6	4,50	35,49						
7	5,00	43,47						
8	5,50	52,14						
9	6,00	61,49						
10	6,50	71,46						

Using the above data, one can calculate the values of the coefficient of friction μ, the coefficient of the drag force C, the maximum attainable speed v_{max}, of the skier (assuming that he starts his descent from a sufficient high altitude).

A. (7 points) Using suitable new variables, present the dependence of the skier motion on the experimental results in a way that can be studied graphically and the information on the parameters describing the motion to be extracted from the graph, such as the drag coefficient C, the coefficient of the frictional force μ and the maximum attainable speed, v_{max}. Give the expressions for the new variables and for the parameters of the used functional dependence as functions of the given parameters α, g, m, ρ, A.

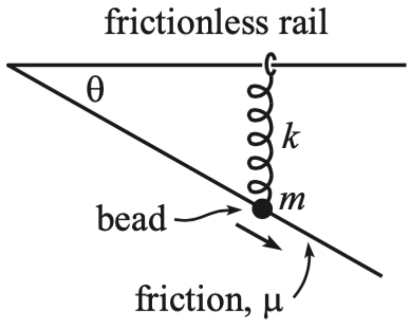
B. (4 points) Calculate the values of the new variables for each measurement based on the given data and fill the empty columns of the table.

C. (4 points) Draw the dependence in the graph paper provided.

D. (10 points) Using the graph, calculate the values of the coefficient of friction μ, the drag coefficient C and the maximal attainable speed of the skier v_{max} (if the hill is sufficiently high).

Problem 3: "The bead on the spring"

Assume there is a setup such as the one shown in the next figure. A bead with mass m is constrained to move along a rail which is connected with another rail at an angle θ. The rail has friction and the coefficient of the kinetic friction with the bead is μ. The bead is connected with a spring of spring constant k. The other end of the spring is constraint to move on a frictionless rail. The spring is always perpendicular to the rail due to the fact the rail is frictionless and the spring does not have a mass. The relaxed length of the spring is zero. Assume also that the system is not affected by gravitational effects.



The bead starts to move at the vertex of the rails and the spring is at its relaxed length of zero. An impulse is given to the system so it starts to move with velocity $\vec{v}_0$ along its rail. The figure shows the configuration of the system at a time $t > 0$.

A. (**6 points**) Draw the free body diagram for the bead at an arbitrary moment and give the mathematical expressions for the forces you draw. If x is the distance the bead has travelled along its rail, determine all the forces and express them in terms of this distance x and the other constants.

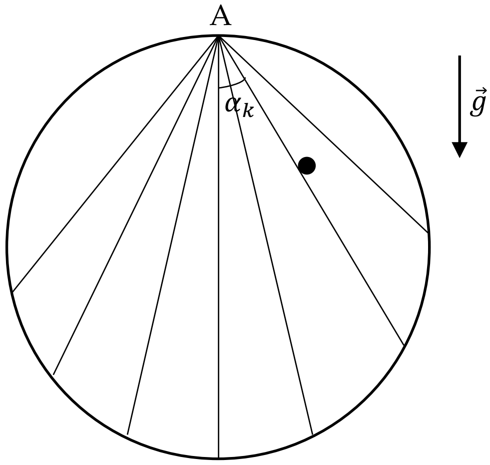
B. (**8 points**) Derive the expression for the distance the bead is covering before it comes to a stop for the first time.

C. (**5 points**) Determine the condition for the bead to start moving again. Specify the direction the bead is going to move. Assume that the coefficient of static friction is also μ.

D. (**6 points**) Assume that the bead really does move towards the vertex of the rails. Determine its speed when it arrives back to the vertex.

Problem 4: Galileo Galilei – "The minimum descent time"

This year the world of Physics commemorates 380 years since Galileo Galilei (1564-1642), one of the founders of modern science, passed away. During the final years of his life, Galileo approached an interesting problem dedicated to the "minimum descent time", or, what is today called the "Galilei's problem".

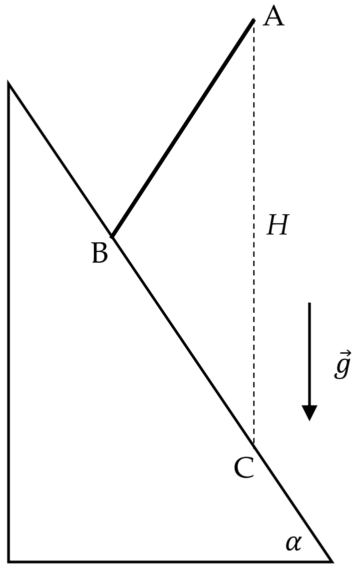


A. (5 points) From a point A, with zero initial speed, a big number of identical material points move in a vertical plane, in a uniform gravitational field (the gravitational acceleration, g, is known), sliding without friction along ramps, inclined at different angles to the vertical (*see* the adjacent figure). If the point A lies at the highest point of a vertical circle of radius R, determine:

A1. (2 points) on which of the ramps the material points reach back to the circle in the shortest time;

A2. (3 points) the geometric locus of the points occupied by all the material points at any given time during their motion on the ramps (*i.e.*, the geometrical figure described by the moving material points, at any given moment of time, if a snapshot is taken).

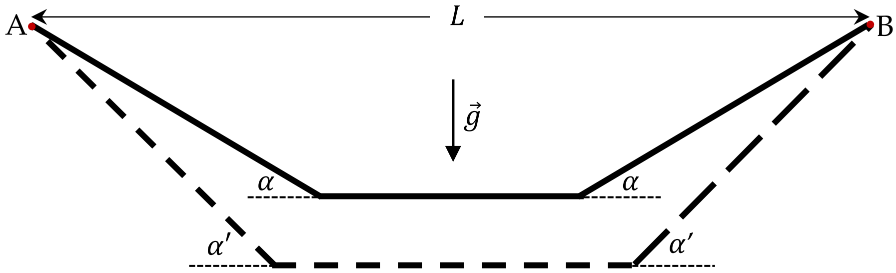
B. (9 points) An inclined plane makes the angle $\alpha = 60°$ with the horizontal plane (*see* the figure below). A point A is situated above the inclined plane, at the height $H = \mathrm{AC} = 29{,}43\ \mathrm{m}$, measured along the vertical direction, above the inclined plane.



B1. Find the position of a point B on the inclined plane (*i.e.*, the mathematical expression for the length of the segment BC), such that a material point, leaving A with zero initial speed, and sliding without friction along the rectilinear ramp AB, to arrive in B in the shortest time. The gravitational acceleration is $g = 9{,}81\ \mathrm{m/s^2}$ and has the same value in all the points. Calculate the numerical value for the length of the segment BC.

B2. Determine the mathematical expression for the minimum descent time on the ramp AB and calculate its numerical value.

C. (11 points) In his last book[1], in which motion is treated mathematically for the first time in history, Galileo analyzes the problem of the fastest descent in uniform gravitational field without friction and concludes that the arc of a circle is faster than any number of its chords. Although it was later proved that the circle is not the path of fastest descent, Galilei admits that any possible error will be removed after future advances in Mathematics. The problem of fastest descent (also called the brachistochrone problem) – formulated in 1696 by Johann Bernoulli and correctly solved by his older brother Jakob Bernoulli – actually shows that the curve is different from a circle and is called cycloid, a name given by Galilei himself.



Since the brachistochrone problem is complicated, it can be simply modeled by three connected rectilinear ramps, the descent time surpassing the cycloidal one with just a few percent. To further simplify the model, a symmetric arrangement will be considered, as illustrated in the figure above. A material point is released in A, with zero initial speed and slides on the ramps without friction from A to B. The middle ramp is horizontal and the transition from one ramp to the next is smooth. The horizontal distance between the fixed points A and B is $L = 5{,}66$ m.

Note: *By modifying the angle α, while keeping the points A and B fixed, the lengths of all the ramps will change. For an angle $\alpha' > \alpha$, the new configuration is represented with dashed lines on the above figure.*

C1. (8 points) Determine the angle α of the inclined ramps such that the transition time τ from A to B to be minimum.

C2. (2 points) Derive the expression for the length of the entire path of the material point, L_{tot}, when the transition time is minimal and calculate its numerical value.

C3. (1 points) Derive the mathematical expression for the minimum transition time τ_{min} and calculate its numerical value.

[1] Galileo Galilei, *Discourses and Mathematical Demonstrations Relating to Two New Sciences*, 1638, Leiden, South Holland.