Problem 1 - City car

A typical city car of a mass $m = 1000$ kg is equipped with a motor of a maximum output power $P_{max} = 80$ kW. Only the front wheels are leading, i.e. propelled by the motor, while the rear wheels rotate freely, with negligible friction. The center of mass of the car is midway between the front and the rear wheels and midway between the left and the right wheels. The coefficient of static friction between the wheel tires and the road is $\mu = 0.8$. Assume that the acceleration due to gravity is $g = 10\ \mathrm{m/s^2}$. In all parts of the problem the car moves on a horizontal road in a straight line.

a) **(4 points)** What is the maximum acceleration a_{max} of the car, when starting from rest?

b) **(14 points)** What is the minimum time t_{min} necessary to accelerate the car from rest to a velocity $v_{\mathrm{f}} = 30\ \mathrm{m/s}$?

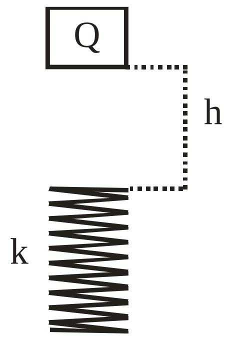
c) **(7 points)** As the velocity surpasses v_{f}, the further motion of the car is affected significantly by the air drag force F_{d}, which was neglected in points **a)** and **b)**. Its magnitude is proportional to the square of the velocity v of the car:

$$F_{\mathrm{d}} = Dv^2$$

where $D = 0.5$ kg/m is a drag coefficient. By using this information find the maximum velocity v_{max}, which could be attained by the car.

Problem 2 – Block on a spring

Consider a block weighting $Q = 4000$ N is maintained at rest at a height $h = 5$ m above an ideal spring. The spring has a spring constant $k = 8000$ N/m and is initially in its equilibrium state. The block is released and falls towards the spring while subjected to a friction constant force $F_0 = 1000$ N due to air resistance.



a) **(3 points)** Determine the expression for compressing distance x_0 of the spring after the block falls on it, neglecting the friction force, and calculate its numerical value.

b) **(4 points)** Taking into account the friction force, determine the expression for the compressing distance x_1 of the spring after the block falls on it and calculate its numerical value.

c) **(4 points)** Taking into account the friction force, determine the expression for the maximum height h_1 of the block after its first contact with the spring (*i.e.* after the first „jump“) and calculate its numerical value.

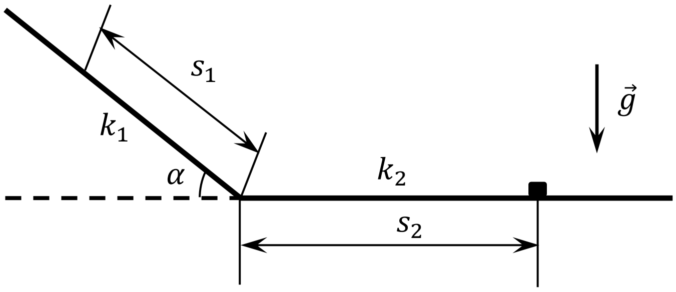
d) **(6 points)** Taking into account the friction force, determine the expression for the maximum height h_2 of the block after its second contact with the spring (*i.e.* after the second „jump“) and calculate its numerical value.

e) **(5 points)** Taking into account the friction force, will the block jump after the third contact between the block and the spring? Explain your answer.

f) **(3 points)** Taking into account the friction force, calculate the final compressing distance x_f of the spring after the block comes to rest.

Problem 3 - Friction Coefficients

A body of negligible size slides down on an inclined plane. The distance between the initial position of the body (where it was at rest) and the base of the inclined plane is s_1. The angle between the inclined plane and a horizontal plane is α. The friction coefficient between the body and the inclined plane is k_1, and the one between the body and the horizontal plane is k_2, respectively. At the end of its motion the body stops at distance s_2 from the base of the inclined plane (from the beginning of the horizontal plane, see the figure). There is no energy loss at the transition of the body from the inclined to the horizontal plane.



$\alpha/\ ^\circ$	s_2/ cm			
15	8.0			
20	19.5			
25	30.3			
30	40.6			
35	51.0			
40	61.4			
45	70.5			
50	79.9			
55	88.3			
60	95.5			

a) **(4 points)** Derive a formula expressing the dependence of distance s_2 on the given parameters (α, s_1, k_1, and k_2).

b) **(3 points)** At a fixed distance $s_1 = 0.500$ m it was experimentally investigated the dependence of the distance s_2 on the angle α. The measurements are tabulated in the table. Using suitable new variables present the experimental dependence in such a way that it can be easily processed graphically to determine the friction coefficients. Give the expressions of the new variables as functions of the given parameters.

c) **(9 points)** Using the empty columns in the table, calculate the values of the new variables for each measurement. Draw the dependence on the graph paper provided.

d) **(6 points)** Using the obtained graph calculate the values of the friction coefficients k_1 and k_2.

e) **(3 points)** From the obtained data calculate the value of the maximal angle α_{max} at which the body will still stand at rest on the inclined plane.

For **problem 3**

Problem 4 – Electric resistances measurement

A. For measuring resistances, a student makes the electric circuit illustrated in Fig. 1. The switch k can be commuted in positions labeled with 1, respectively 2 in the diagram. R is the unknown value of the resistance, V is the voltmeter and A is the amperemeter. The internal resistance of the voltmeter is R_V and of the amperemeter is R_A.

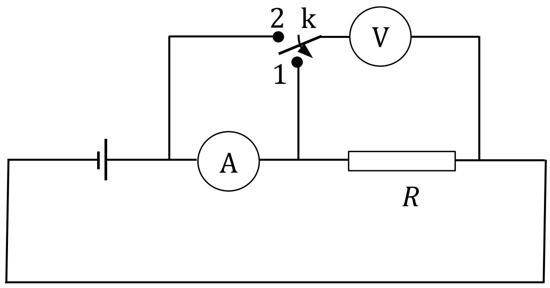


Fig. 1

a) **(4 points)** Derive, for each position of the switch (1, respectively 2), the measured value of the resistance (R_m), as a function of R, R_V, and R_A.

Because the amperemeter and the voltmeter are not ideal, the measured values R_m of the resistance are different from the true value R of the resistance.

b) **(1 point)** Derive the expression of the relative variation $\alpha_R = \frac{R_m - R}{R}$ (as a measure of accuracy) for each position of the switch.

Depending on the value of the resistance R, a certain position of the switch is preferred over the other.

c) **(6 points)** Based on the results obtained above, derive the range of values for the resistance R for which each position of the switch is preferred. Express the results in terms of R_A and R_V. Using the well-known relationship between R_A and R_V, prove that the (practical) approximate mathematical expression for the limit between the two resistance ranges is $\beta\sqrt{R_A R_V}$ and derive the numerical value for the coefficient β.

B. Using some of the circuit elements illustrated in Fig. 1, the student is assembling their own ohmmeter. The purpose is to measure an unknown resistance R_x of a resistor. The result is shown in Fig. 2. The e.m.f. of the generator is $E = 9.00$ V (its internal resistance is neglected) and the internal resistance of the amperemeter is $R_A = 10.0\ \Omega$. The

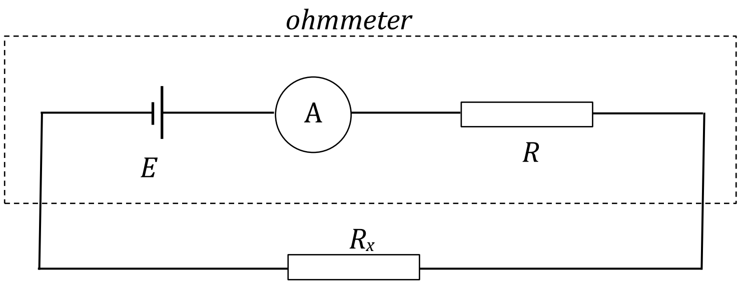


Fig. 2

amperemeter is an analog one, with the scale range of $I_{max} = 50$ mA and with $N = 100$ divisions (Fig. 3). The current is indicated by a needle.

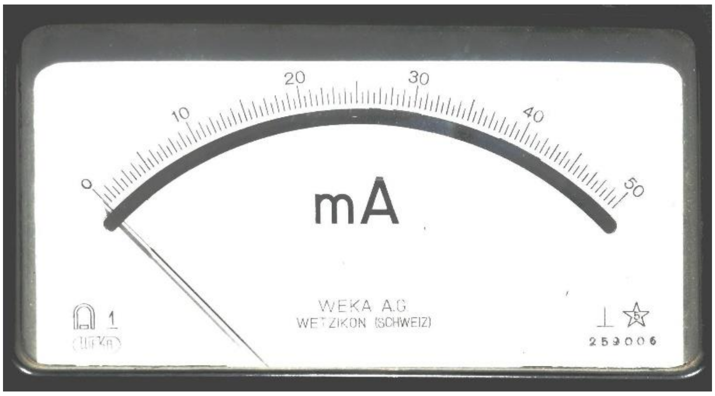


Fig. 3

The student chooses the resistor with the resistance R such that, when their ohmmeter is short-circuited, the needle indicates the maximum value on the scale.

d) **(2 points)** Derive the mathematical formula for R and calculate its numerical value.

When the resistor with the unknown resistance is connected to the ohmmeter, the needle deviation is n divisions.

e) **(1.5 points)** Express R_x as a function of $n, N, E,$ and I_{max}.

The student measures the resistances of several resistors and wonders for which of them the precision is the best possible.

f) **(4 points)** Derive the mathematical formula for the relative error of determining R_x. Based on this result, derive the value of n for which this relative error is minimum.

Note: consider that $\left(\frac{\Delta n}{n}\right)^2 \ll 1$, *where* Δn *is precision with which the student reads the number* n *of divisions on the amperemeter scale.*

g) **(3.5 points)** Obtain the expressions for R_x and for the absolute error ΔR_x, using n derived at question **f**) and calculate their numerical values.

During the experiments, the student did not place the amperemeter in front of them, but to their right, such that the angle between the reading direction and the perpendicular to the scale is θ. The length of the amperemeter scale is L and the distance between the needle and the scale's plane is h.

h) **(1.5 points)** Obtain the expression for the systematic error caused by the incorrect reading of the current.

i) **(1.5 points)** Calculate the maximum value of the angle θ such that reading error to be less than one division on the amperemeter scale. The length of the amperemeter scale is $L = 10$ cm and the distance between the needle and the scale's plane is $h = 2.0$ mm.