1st Balkan Physics Olympiad – 2019 BPO
July 14-18, Thessaloniki, Greece

Problem 1 - Gravitational Billiard

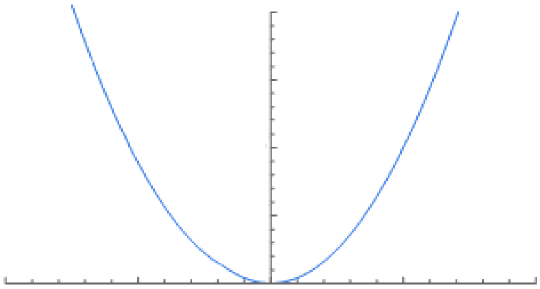
A point particle is moving in a homogeneous gravitational field close to the Earth surface in a vertical plane limited from below by a parabola (as shown in the figure). Consider periodic motion in a gravitational billiard. A periodic motion repeats itself indefinitely when friction forces are absent. After reflection from the boundary the energy of the particle remains unchanged.

a) Draw the periodic trajectories that touch the boundary in one, two and three distinct points. **[3x3 points]**

b) Calculate the period for the first two cases. [One touching point - **6 points**]

[Two touching points - **10 points**]

The angle α between the tangent of the parabola $y = ax^2 + bx + c$ at an arbitrary point x_0 and the x-axis is given by $\tan \alpha = 2ax_0 + b$.



Problem 2

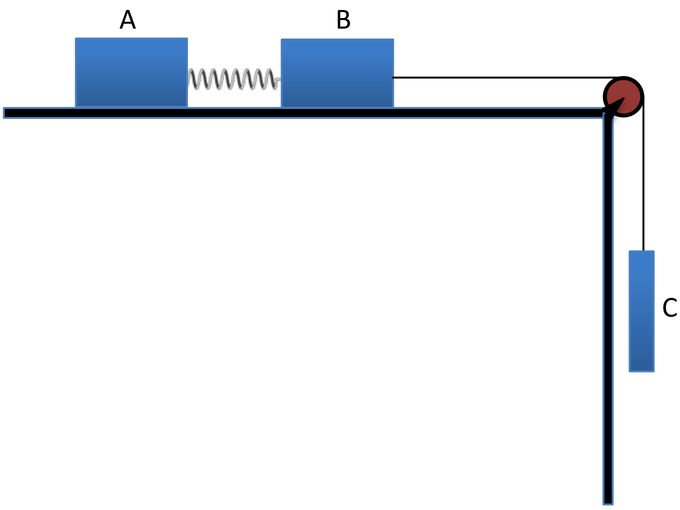
Two identical bodies A and B lie on a smooth horizontal surface. The two bodies are connected by a spring, with negligible mass, that obeys the Hooke's Law, with spring constant k and relaxed length L_0. A third body C, is suspended to the body B, by means of an ideal string (non-extensible and of negligible mass), passing over a smooth pulley P. The three bodies have the same mass m. Initially, the three-body system is at rest and the spring has its relaxed length. The system is accordingly released from rest.

a) Draw a free body diagram for each body after the system is released **[3 points]**

b) Calculate the minimum and maximum distance between the bodies A and B. Mind that during the three-body motion, body B doesn't hit on the pulley and body C doesn't reach the ground. **[10 points]**

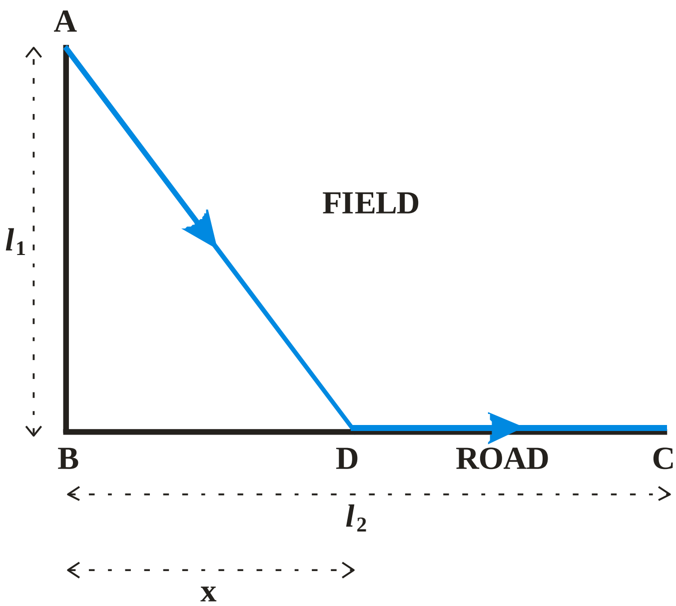
c) Calculate the minimum value for the friction coefficient as well as the distance between the bodies A and B, in order for the system to remain at rest when it is released. **[6 points]**

d) Calculate the value of the friction coefficient as well as the distance between the bodies A and B in order for the system to move at constant speed. **[6 points]**



Problem 3

A man stands at a point A in a field, at a distance $l_1 = 600\,m$ from the road BC. In the field his velocity is $v_1 = 1\,m/s$, while on the road BC it is $v_2 = 2\,m/s$. He can walk in the field along AD and on the road along DC in order to reach the final destination C. The distance between B and C is $l_2 = 800\,m$. Your final assignment is to find his route, so that he can reach the destination in the least possible time and to determine the time elapsed. In order to do this, you need to find the coordinate of the point D, i.e., the distance $x = BD$. We will lead you through several steps in order to fulfill this assignment.



a) Find the expression for the time t_1 for the motion on the field (route AD) as a function of x. **[2 points]**

b) Find the expression for the time t_2 for the motion on the road (route DC) as a function of x. **[2 points]**

c) Find the expression for the total time t as a function of x. **[2 points]**

d) Let your solutions for the total time t and the position x be $t = t_0$ and $x = x_0$. Put these solutions in your main expression for the total time. Using elementary algebraic manipulations (squaring, etc.) you should get rid of all the square root(s) and fraction(s) in your expression. **[3 points]**

e) Now, if the solution for x is a little bit different from the real solution x_0 (in other words, if we put the point D a little bit closer to B or to C), then the solution for t would also be a little bit different from the real solution t_0. So, we need to try to "shake" the solutions to see if they really are the correct solutions. To do this, we suggest you to add small temporal and spatial displacements (perturbations) Δt and Δx, *i.e.*, instead of t_0 and x_0 put $t_0 + \Delta t$ and $x_0 + \Delta x$ in your main expression obtained in the previous step. **[3 points]**

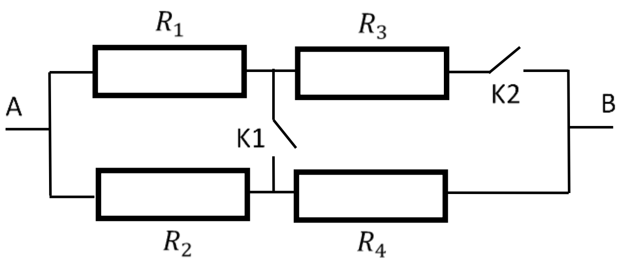
f) Rewrite the expression from the previous step neglecting all terms containing very small products, such as $\Delta t \cdot \Delta t$, $\Delta t \cdot \Delta x$ and $\Delta x \cdot \Delta x$. **[4 points]**

g) Using the result from the previous step, express the ratio between small temporal and spatial displacements Δt and Δx, *i.e.*, express the quantity $\Delta t / \Delta x$. **[3 points]**

h) Your final job in this assignment is to calculate the route and the total time. To do this, equate $\Delta t / \Delta x$ to zero, and with the help of some previous steps you will be in the position to proceed towards obtaining x_0 and t_0. **[6 points]**

Problem 4

An electric circuit consists of four resistors and two switches (see the figure below). Let's denote the state of the switch with "0" if it is opened, and with "1" if it is closed (in the figure both switches are opened but this may not be true for the initial state of the circuit). The initial states of the switches are unknown. The resistance R_{AB} between the two ends A and B of the circuit is measured. Its initial resistance (state "α" of the circuit) is $R_{\mathrm{AB\alpha}} = 240\ \Omega$. The switch K1's state is reverted and the resistance of the circuit remains the same, $R_{\mathrm{AB\beta}} = 240\ \Omega$. After that, the switch K2's state is reverted and the resistance of the circuit becomes $R_{\mathrm{AB\gamma}} = 400\ \Omega$. Finally, the switch K1's state is once again reverted and the resistance of the circuit becomes $R_{\mathrm{AB\delta}} = 280\ \Omega$.



From the available experimental data, calculate the values of the resistances of the four resistors in the circuit and the initial state of each of the two switches. Fill your answer in the table below.

State of the circuit	State of K1	State of K2	R_{ABi} Ω
α			240
β			240
γ			400
δ			280

Resistor	Value, Ω
R_1	
R_2	
R_3	
R_4	

Initial states of K1, K2 **[2 x 2,5 points]**

Resistor values **[4 x 5 points]**