EQ1: Official Solution[1]

A.1 (1.0 pt)
Move the magnet along each of the axes and notice the change in the magnet field. For example, if the magnet is aligned along the length of the phone, and you are moving the magnet along the same direction (length of the phone), magnetic field will show the change in sign when the magnet crosses the Magnetometer.

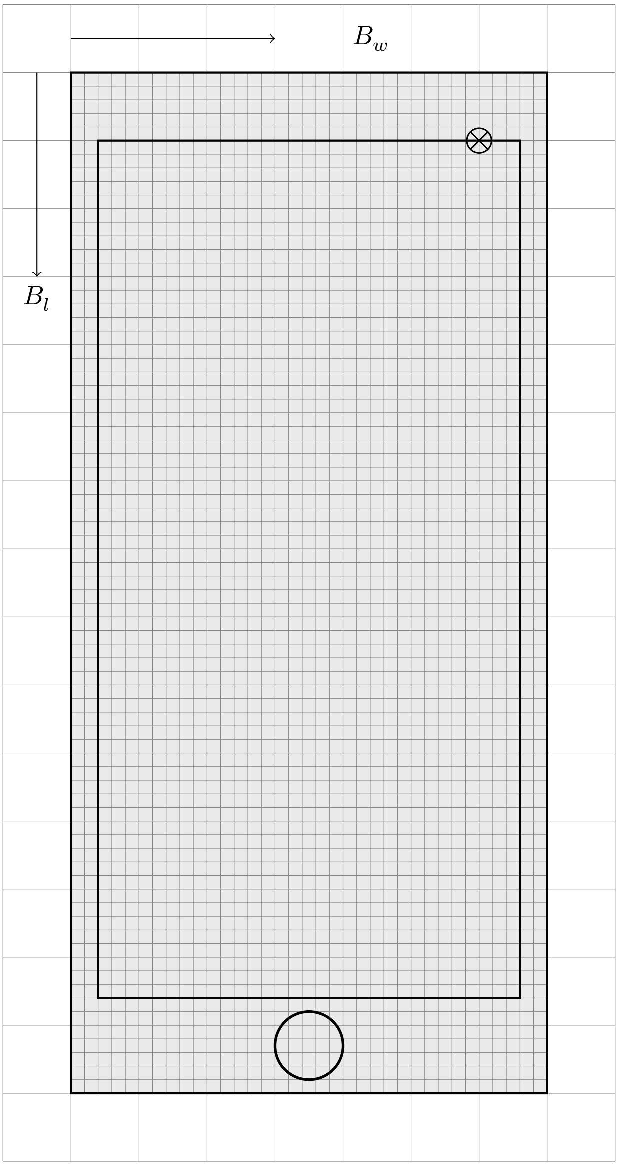
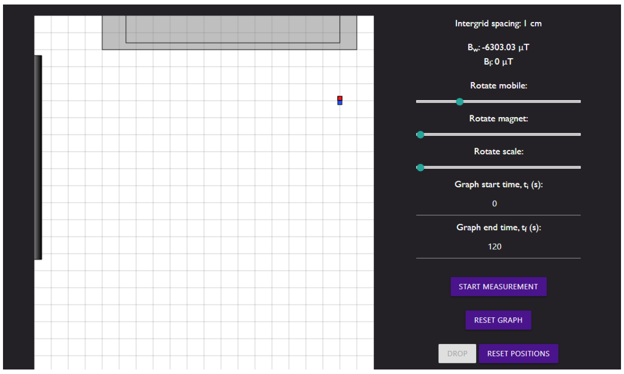


Figure. 1

[1] Chandan Relekar (IISc, Bangalore), Siddhant Mukherjee (The University of Cambridge, UK), Siddharth Tiwary (IIT Powai, Mumbai), Charudutt Kadolkar (IIT Guwahati), Praveen Pathak (HBCSE-TIFR, Mumbai), were the principal authors of this problem. The contributions of the Academic Committee and the International Board are gratefully acknowledged.

A.2 (2.3 pt)
The set up to find Dipole moment



The magnetic field B_w of a point dipole at the distance x $(x >> d)$ from the dipole's center can be approximated by

$$B_w = \frac{\mu_0}{2\pi}\frac{M}{x^3} \tag{1}$$

Rearranging above equation, we get

$$B_w = \frac{\mu_0 M}{2\pi} \times \frac{1}{x^3} \tag{2}$$

From equation (2), a plot of B_w vs $\dfrac{1}{x^3}$ is a straight line passing through origin. Solving the slope will give dipole moment of the magnet.

A.2 (cont.)

Dipole moment of the Magnet:

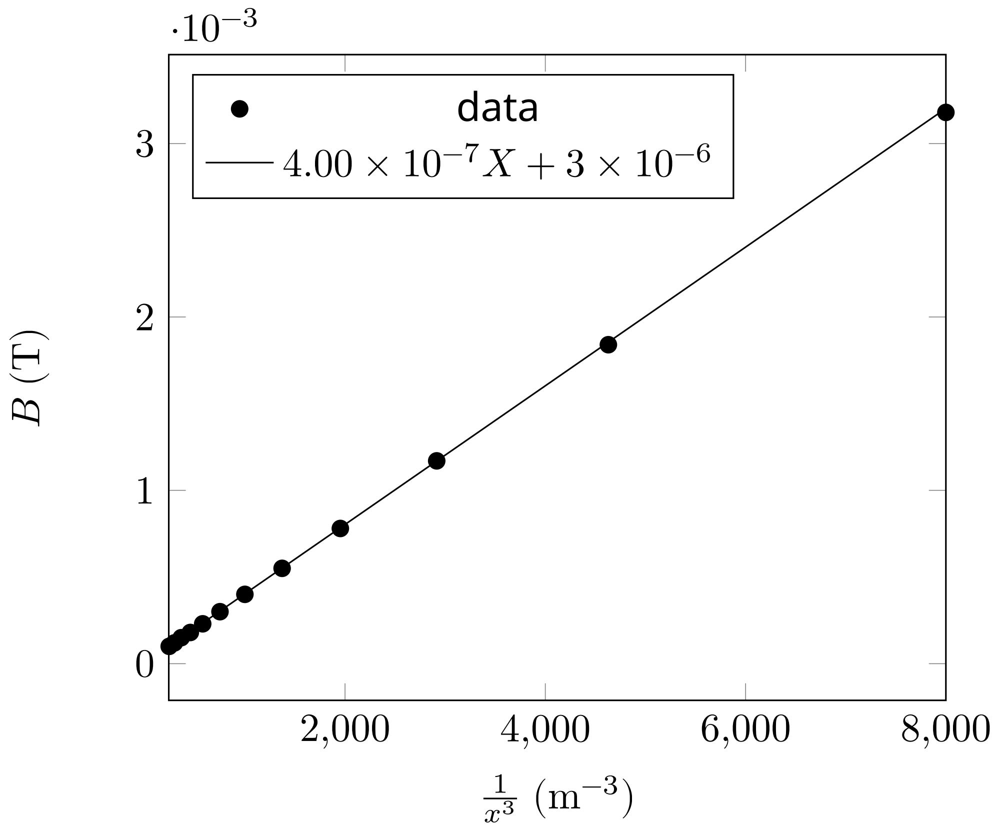
Fill the appropriate quantities.

Sr.No	x (cm)	$B_w(\mu\mathrm{T})$	$\frac{1}{x^3}(\mathrm{m}^{-3})$	B_w (T)
1	5	3177.63	8000	0.00317763
2	6	1841.06	4629.63	0.00184106
3	7	1170.34	2915.45	0.00117034
4	8	783.69	1953.13	0.00078369
5	9	550.22	1371.74	0.00055022
6	10	403.42	1000	0.00040342
7	11	301.21	751.31	0.00030121
8	12	231.96	578.7	0.00023197
9	13	181.58	455.17	0.00018158
10	14	146.03	364.43	0.00014603
11	15	118.72	296.3	0.00011872
12	16	98.18	244.14	0.000099

From equation (2), a plot of B_w vs $\dfrac{1}{x^3}$ is a straight line passing through the origin. The magnitude of the dipole moment can be calculated from the slope.

A.2 (cont.)

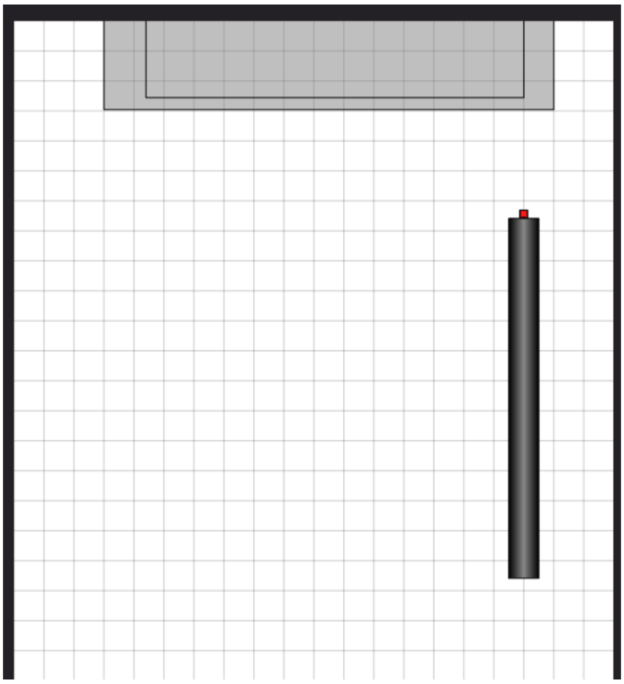
Graph of $\boxed{B_w}$ vs $\boxed{\frac{1}{x^3}}$



Slope of the graph: $4.00 \times 10^{-7}\ \mathrm{Nm^2/A}$

Dipole moment of the magnet: 2.00 $\mathrm{Am^2}$

B.1 (0.3 pt)
Rotate the smartphone and align the magnet and the pipe along the width of the phone as shown below.



Consider the case when the magnet is at rest at a distance x_0 from the magnetometer origin. The magnet is released along the axis of the pipe. It will start descending through the pipe. In the conducting sections of the pipe, after a brief period of accelerated motion, the magnet will attain a terminal velocity v, due to the presence of eddy current damping. In this case, the magnetic field B_w measured by the magnetometer changes with time t as

$$B_w(t) = \frac{\mu_0}{2\pi}\frac{M}{(x_0+vt)^3} \tag{3}$$

Equation (3) is rearranged as

$$\left(\frac{\mu_0 M}{2\pi B_w(t)}\right)^{1/3} = vt + x_0 \tag{4}$$

B.1 (cont.)

Obtained profile of magnetic field vs time clearly suggests three distinct phases (AB, BC, and CD) of the magnet's motion (see Fig. 4 below).

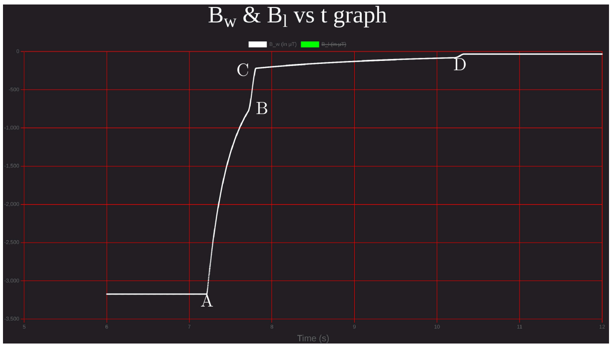


Figure (4)

We collect the data of (B_w vs t) for all three phases and plot them according to Eq. (4). For the acceleration phase of the pipe (wooden section), the graph will be non-linear and for the conducting pipe sections (Al and Cu) where the magnet moves with the terminal velocities, the graph will be linear. Duration of accelerated motion before attaining terminal velocity in the conducting sections of the pipe may be neglected.

It can be clearly seen from the graphs in the next sub parts that:

Section	Section number
Aluminium	1
Copper	3
wood	2

Since the copper has higher conductivity than aluminium, the terminal velocity in Cu section will be lower than Al section.

B.2 (2.6 pt)
Terminal velocity of Magnet in aluminium section:
Fill the appropriate quantities.

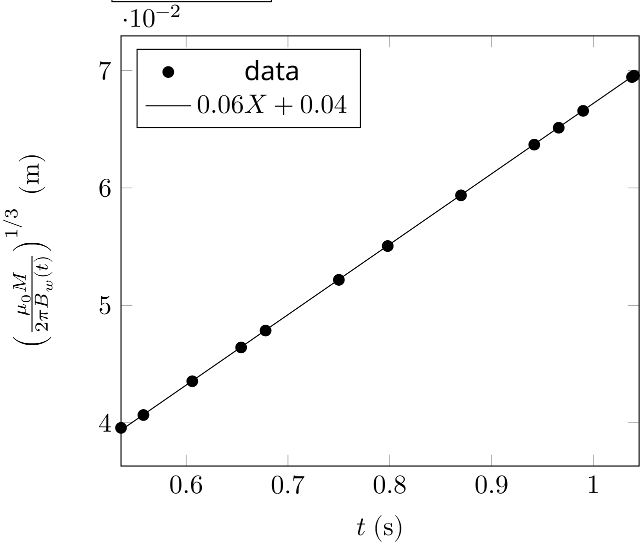
Sr.No	B_w (μT)	t (s)	$\left(\frac{\mu_0 M}{2\pi B_w(t)}\right)^{1/3}$ (m)	v (m/s)
1	6462.28	0.534	0.0396	0
2	6462.28	0.536	0.0396	0.01
3	5954.24	0.558	0.0407	0.06
4	4850.05	0.606	0.0435	0.06
5	4002.02	0.654	0.0464	0.06
6	3651.48	0.678	0.0478	0.06
7	2817.43	0.75	0.0522	0.06
8	2397.97	0.798	0.055	0.06
9	1911.68	0.87	0.0594	0.06
10	1548.47	0.942	0.0637	0.06
11	1448.01	0.966	0.0651	0.06
12	1356.06	0.99	0.0666	0.06
13	1194.26	1.038	0.0694	0.06
14	1188.09	1.04	0.0696	0.14
15	1142.95	1.05	0.071	

The velocity (v) column is obtained using the forward difference $\dfrac{x_{n+1}-x_n}{t_{n+1}-t_n}$.

From equation (4), a plot of $\left(\dfrac{\mu_0 M}{2\pi B_w(t)}\right)^{1/3}$ vs t will be a straight line. The slope of the line will give terminal velocity .

B.2 (cont.)

Graph of $\boxed{\left(\frac{\mu_0 M}{2\pi B_w(t)}\right)^{1/3}}$ vs $\boxed{t}$



Slope of the graph: $6\ \mathrm{cm/s}$.

Terminal velocity of the magnet in aluminium section: $6\ \mathrm{cm/s}$

Length of the aluminium section:

Slope of the graph: Not used	Intercept of the graph: Not used

Values in the fourth column in the above table actually represents the distance of the magnet from the magnetometer. We calculate the velocity (column 5) of the magnet using the forward difference method. Notice that the magnetic field starts to change at 0.536 s. Hence this is the point at which the magnet starts descending. It quickly achieves the terminal velocity of $6\ \mathrm{cm/s}$. A change in velocity (or departure from the flat line in v vs t curve, if you plot) occurs at 1.040 s. This suggests that the magnet moved out of the aluminium section of the pipe at this time.

Length of the Al section $= (1.040 - 0.536) \times 6\ \mathrm{cm} = 3.024\ \mathrm{cm}$.

B.3 (2.2 pt)
Terminal velocity of magnet in copper section:
Fill the appropriate quantities.

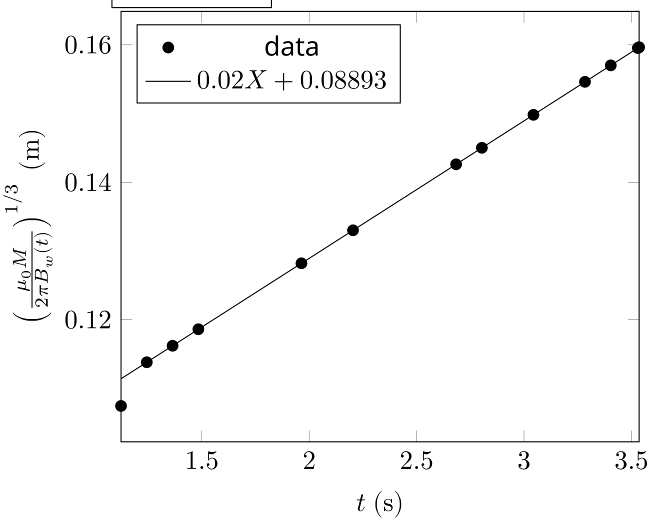
Sr.No	B_w (μT)	t (s)	$\left(\frac{\mu_0 M}{2\pi B_w(t)}\right)^{1/3}$ (m)	v (m/s)
1	338.23	1.122	0.106	0.85
2	322.39	1.124	0.1075	0.87
3	271.32	1.244	0.1138	0.02
4	254.86	1.364	0.1162	0.02
5	239.70	1.484	0.1186	0.02
6	189.79	1.964	0.1282	0.02
7	169.97	2.204	0.1330	0.02
8	137.91	2.684	0.1426	0.02
9	131.17	2.804	0.1450	0.02
10	118.96	3.044	0.1498	0.02
11	108.22	3.284	0.1546	0.02
12	103.34	3.404	0.1570	0.02
13	98.52	3.53	0.1595	0.02
14	98.44	3.532	0.1596	0.02
15	98.37	3.534	0.1596	0.02
16	98.30	3.536	0.1597	0.05
17	98.22139	3.538	0.1597	

The velocity (v) column is obtained using the forward difference $\dfrac{x_{n+1}-x_n}{t_{n+1}-t_n}$.

From equation (4), a plot of $\left(\dfrac{\mu_0 M}{2\pi B_w(t)}\right)^{1/3}$ vs t will be a straight line. The slope of the line will give the terminal velocity.

B.3 (cont.)

Graph of $\boxed{\left(\frac{\mu_0 M}{2\pi B_w(t)}\right)^{1/3}}$ vs $\boxed{t}$



Slope of the graph: $2\ \mathrm{cm/s}$.

Terminal velocity of magnet in copper section: $2\ \mathrm{cm/s}$

Length of copper section:

Slope of the graph: Not used

Intercept of the graph:Not used

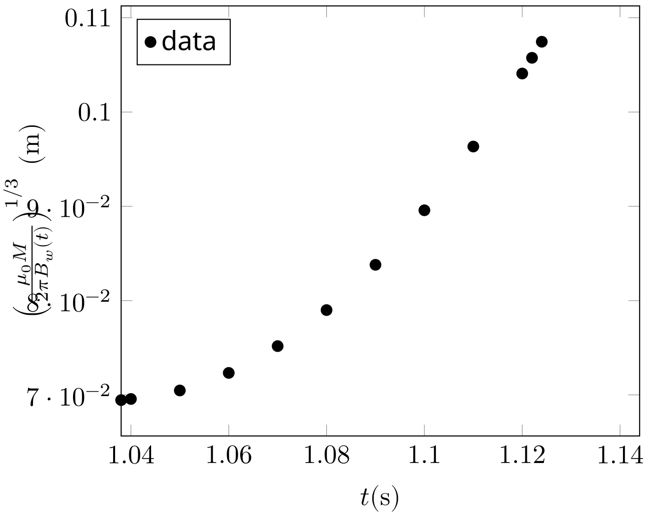
Length of the copper section of the pipe can be calculated by $v_c t_c$, where v_c is the terminal velocity of the magnet in the copper section of the pipe and t_c is the time spent by the magnet in the copper section of the pipe.
Data in the above table suggests that the magnet enters the copper section at $1.124\,\mathrm{s}$, and it leaves the copper section of the pipe at $3.536\,\mathrm{s}$ where we see sudden velocity change.
Length of the copper section $= (3.536 - 1.124) \times 2\ \mathrm{cm} = 4.824\,\mathrm{cm}$.

B.4 (1.6 pt)
Length of wooden section:
Fill the appropriate quantities.

Sr.No	B_w (μT)	t (s)	$\left(\frac{\mu_0 M}{2\pi B_w(t)}\right)^{1/3}$ (m)	v (m/s)	a (m/s^2)
1	1194.26	1.038	0.0694	0.06	4
2	1188.09	1.04	0.070	0.14	5.66
3	1142.95	1.05	0.071	0.15	9.8
4	1056.94	1.06	0.072	0.25	9.8
5	941.55	1.07	0.075	0.34	9.8
6	811.40	1.08	0.079	0.44	9.8
7	679.75	1.09	0.084	0.54	9.8
8	556.44	1.1	0.090	0.64	9.8
9	447.31	1.11	0.096	0.74	9.8
10	354.74	1.12	0.104	0.83	9.8
11	338.23	1.122	0.106	0.85	-334.92
12	322.39	1.124	0.108	0.18	0.31
13	271.32	1.244	0.114		

The velocity (v) and acceleration (a) columns are obtained using the forward difference $\frac{x_{n+1} - x_n}{t_{n+1} - t_n}$ and $\frac{v_{n+1} - v_n}{t_{n+1} - t_n}$ respectively.

B.4 (cont.)



Wooden section is the middle section of the pipe. We have already established in Al section that the magnet exits the Al section at 1.040 s. We tabulate the data of velocity vs time for this section. Notice that the velocity of the magnet suddenly drops at 1.124 s. At this moment, the magnet enters in the copper section of the pipe and comes under the influence of damping due to the eddy current.
Length of the pipe can be calculated by $\left(v_{\mathrm{Al}} t_w + \frac{g t_w^2}{2}\right)$, where v_{Al} is terminal velocity of magnet in the aluminium section of the pipe and t_w is the time spent by the magnet in the wooden section of the pipe.
Total time spent by the magnet in the wooden pipe $(t_w) = (1.124 - 1.040)\ \mathrm{s} = 0.084\ \mathrm{s}$
Length of the wooden section of pipe $= 3.96\ \mathrm{cm}$

Section	EQ1: Magnetic black box marking scheme	Partwise marks	Total marks
A1	Identifying correct location of the inbuilt sensor (full marks if both the coordinates correct within ± 0.2 cm, else no credit	**1.0**	**1.0 pt**
A2			**2.3 pt**
	Choice of variables for plotting	**0.1**	
	Units on these variables	**0.1**	
	For observations of x and B	***(0.8)***	
	less than 5 readings	0	
	5 readings	0.5	
	6 readings	0.6	
	7 readings	0.7	
	8 or more readings	0.8	
	Calculation of quantities to be plotted (full marks only if calculations are correct for all points, else no credit)	**0.1**	
	Graph	**(0.8)**	
	Choice of Scale (minimum 70% coverage)	0.2	
	Both axes labelled with proper units	0.2	
	At least 8 points plotted correctly	0.3	
	At least 6 points plotted correctly	0.1	
	Less than 6 points plotted	0	
	Best fit	0.1	
	Value of dipole moment (Deduct 0.1 for wrong or no unit)	**(0.4)**	
	Between 1.9 and 2.1 Am^2	0.4	
	1.8 to less than 1.9 and greater than 2.1 upto 2.2 Am^2	0.2	

Section		Partwise marks	Total marks
B1	Predicting the sections in correct order	**0.3**	**0.3 pt**
B2			**2.6 pt**
	Schematic diagram of set up (exact location and orientation)	**0.1**	
	Choice of variables for plotting	**0.3**	
	For observations of t and B	**(0.3)**	
	less than 7 readings	0	
	7 to 9 readings	0.2	
	10 or more readings	0.3	
	Calculation of quantities to be plotted (full marks only if calculations are correct for all points, else no credit)	**0.3**	
	Graph	**(1.0)**	
	Choice of Scale (minimum 70% coverage)	0.2	
	Both axes labelled with proper units	0.1	
	At least 10 points plotted correctly	0.5	
	8 or 9 points plotted correctly	0.4	
	6 or 7 points plotted correctly	0.3	
	Best fit	0.2	
	Value of terminal velocity (Deduct 0.1 for wrong or no unit)	**(0.2)**	
	Between 5.9 and 6.1 cm/s	0.2	
	5.8 to less than 5.9 cm/s OR greater than 6.1 upto 6.2 cm/s	0.1	
	Determination of length of aluminium section	**(0.4)**	
	Between 2.9 and 3.1 cm	0.4	
	2.8 to less than 2.9 cm OR greater than 3.1 upto 3.2 cm	0.2	

B3			2.2 pt
	For observations of t and B	(0.3)	
	less than 7 readings	0	
	7 to 9 readings	0.2	
	10 or more readings	0.3	
	Calculation of quantities to be plotted (full marks only if calculations are correct for all points, else no credit)	**0.3**	
	Graph	**(1.0)**	
	Choice of Scale (minimum 70% coverage)	0.2	
	Both axes labelled with proper units	0.1	
	At least 10 points plotted correctly	0.5	
	8 or 9 points plotted correctly	0.4	
	6 or 7 points plotted	0.3	
	Best fit	0.2	
	Value of terminal velocity (Deduct 0.1 for wrong or no unit)	**(0.2)**	
	Between 1.9 and 2.1 cm/s	0.2	
	1.8 to less than 1.9 cm/s OR greater than 2.1 upto 2.2 cm/s	0.1	
	Determination of length of copper section (Deduct 0.1 for wrong or no unit)	**(0.4)**	
	Between 4.9 and 5.1 cm	0.4	
	4.8 to less than 4.9 cm and greater than 5.1 upto 5.2cm	0.2	

B4 (Method I)	Length of the wooden section of pipe		**1.6 pt**
By drawing graph			
	Choice of variables	**0.1**	
	Units of variables	**0.2**	
	For observations of t and B	**(0.3)**	
	6 to 9 readings	0.2	
	10 or more readings	0.3	
	Calculations of function (determination of acceleration OR Graph plotting)	**(0.6)**	
	If all correct	0.6	
	50% correct	0.3	
	Less than 50% correct	0	
	Determination of length of wooden section (deduct 0.1 for wrong or no unit)	**(0.4)**	
B 4 (Method II) **Using the terminal velocity in Al part, time of free fall and g**	To determine $t = t_w$	**(1.3)**	
	$t = t_w$ *= 0.082 s to 0.086 s*	1.3	
	0.08 s t_w *<0.082 s OR*	0.6	
	0.086 s <t_w *0.088 s*		
	Calculation of L_w as per student's value of t (Deduct 0.1 for wrong or no unit)	**0.3**	
B4 (Method III)	3.9 ≤Length ≤4.1	1.6	**1.6 pt**
By subtracting the lengths			
	Length between 4.2-4.1 and 4.1-4.2 ≤4.1	**0.6**	

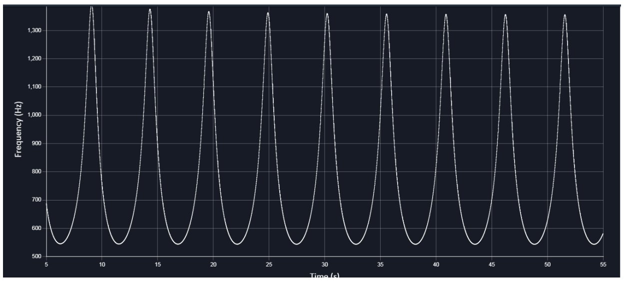
EQ2: Acoustic black box solution[1]

A.1 (0.2 pt)

$$x(t) = v_s t\cos(\beta) + R\cos(\omega t + \phi) + \mathrm{X_C} \tag{1}$$
$$y(t) = v_s t\sin(\beta) + R\sin(\omega t + \phi) + \mathrm{Y_C} \tag{2}$$

A.2 (1.2 pt)

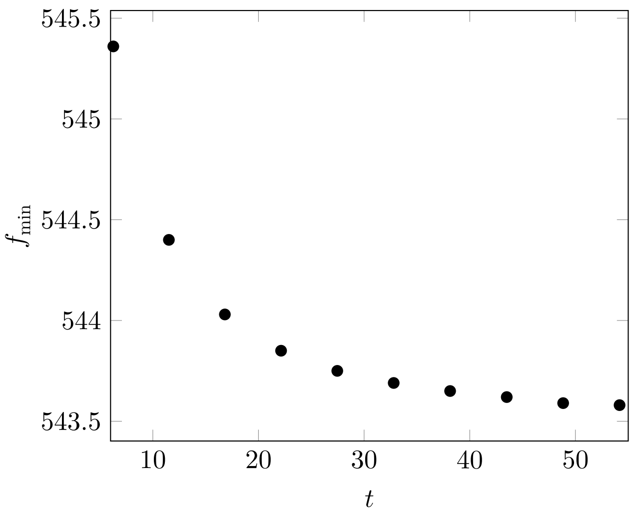
Figure below shows the graph obtained for the data point interval 0.02.



Sr no	t(s)	f_{min}
1	6.26	545.36
2	11.52	544.4
3	16.82	544.03
4	22.14	543.85
5	27.46	543.75
6	32.8	543.69
7	38.14	543.65
8	43.5	543.62
9	48.84	543.59
10	54.18	543.58

[1] Siddharth Tiwary (IIT Powai, Mumbai), Siddhant Mukherjee (The University of Cambridge, UK), Chandan Relekar (IISc, Bangalore), Charudutt Kadolkar (IIT Guwahati), Praveen Pathak (HBCSE-TIFR, Mumbai), were the principal authors of this problem. The contributions of the Academic Committee and the International Board are gratefully acknowledged.

A.2 (cont.)



A.3 (1.0 pt)

We take a general case in which both detector and the source are moving with velocities v_d and v_s respectively. Also, the line joining source and detector makes angle α with the x-axis as defined in fig. 1 of the question.

Note that α is a function of time. Let $\hat{n}$ be the vector joining the source and the detector. For the case when the source is approaching the detector, frequency detected by the detector is

$$
\begin{aligned}
f(t') &= f_0 \frac{c - v_d \cdot \hat{n}(t)}{c - \vec{v}_{\mathrm{T}} \cdot \widehat{n(t)}} && (3)\\
&= f_0 \frac{c - v_d \cos(\gamma - \alpha(t))}{c - [(\vec{v_s} + R\omega\hat{\theta}) \cdot \hat{n}(t)]} && (4)\\
&= f_0 \frac{c - v_d \cos(\gamma - \alpha(t))}{c - [(v_s \cos(\beta - \alpha(t)) + R\omega \cos(\omega t + \phi + \pi/2 - \alpha))]} && (5)\\
&= f_0 \frac{c - v_d \cos(\gamma - \alpha)}{c - [(v_s \cos(\beta - \alpha(t)) - R\omega \sin(\omega t + \phi - \alpha(t))]} && (6)
\end{aligned}
$$

Similarly, for the source moving away from the detector

$$
f(t') = f_0 \frac{c - v_d \cos(\gamma - \alpha)}{c + [(v_s \cos(\beta - \alpha(t)) - R\omega \sin(\omega t + \phi - \alpha(t))]} \quad (7)
$$

The expression of minimum frequency in the asymptotic limit ($t \to \infty$) is

$$
f_{\text{min}} = f_0 \frac{c}{c + [(v_s + R\omega)]} \quad (8)
$$

A.4 (1.4 pt)
Initial location of the source: Keep the detector first on the $x-$axis (say x_1,0°) and then on the $y-$axis (say y_1,90°) and from the graph, note down the time taken to reach the first signal to the detector. Lets denote these timings as Δt_{x1} and Δt_{y1} respectively. Then,

$$(x - x_1)^2 + y^2 = (c\Delta t_{x1})^2 \tag{9}$$

$$x^2 + (y - y_1)^2 = (c\Delta t_{y1})^2 \tag{10}$$

Solving above two equations will give the coordinates of the source. From the simulation, for $x_1 = y_1 = 500$m, $\Delta t_{x1} = 1.5344$ s and $\Delta t_{y1} = 1.2727$ s. Above equations have two solutions. We can keep the detector at third location to choose the correct pair. The answer is

$$x_\mathrm{A} = 419.99, y_\mathrm{A} = 499.99$$

A.5 (2.1 pt)

Let the detector be at such a position where the source approaches the detector from a large distance (say from left side), crosses it and then moves away at a large distance (to the right side). In the asymptotic limits (far left and far right, $\beta \approx \alpha$), two pairs of the frequencies will be detected by the detector. We take $v_d = 0$. On the far left side

$$f_{\max} = f_0 \frac{c}{c - (v_s + \omega R)} \tag{11}$$

$$f_{\min} = f_0 \frac{c}{c - (v_s - \omega R)} \tag{12}$$

On the far right side

$$f_{\max} = f_0 \frac{c}{c + (v_s - \omega R)} \tag{13}$$

$$f_{\min} = f_0 \frac{c}{c + (v_s + \omega R)} \tag{14}$$

Eqs. (11) and (12) yields

$$\frac{f_{\max} + f_{\min}}{f_{\max} - f_{\min}} = \frac{c - v_s}{\omega R} \tag{15}$$

Eqs. (13) and (14) yields

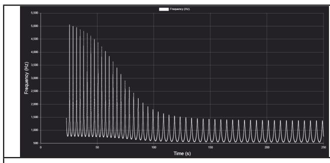
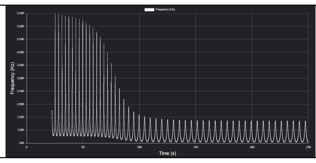
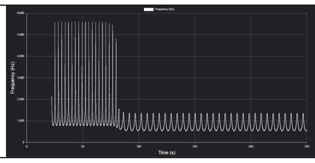
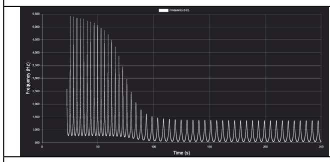
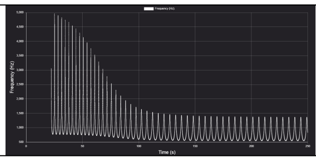
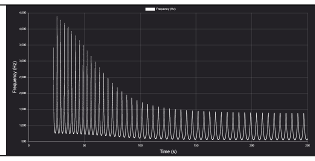
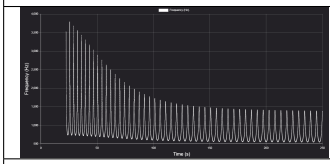
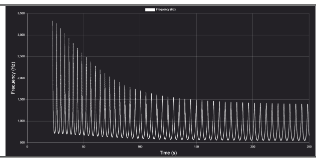
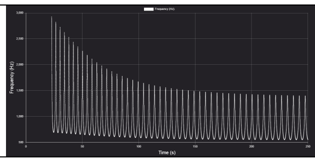
$$\frac{f_{\max} + f_{\min}}{f_{\max} - f_{\min}} = \frac{c + v_s}{\omega R} \tag{16}$$

It is also given that at $t = 0$, there is a finite distance between the source and the detector. This will cause a signal delay. Let Δt be the time interval between two peaks ($f_{\max}$). In this case

$$\Delta t = \frac{2\pi}{\omega}\left(1 + \frac{v_s}{c}\right) \tag{17}$$

Eqs. (15-17) can be solved together to obtain the values of $v_s, \omega,$ and R. It is necessary to keep the stationary detector at such coordinates (say x_D, y_D), so that the source approaches the detector from a large distance, crosses it and then moves away to a large distance. Note that the asymptotic behaviour can be identified in the region where the extrema in the graph remains almost constant. Also, we expect a sharp change in the graph if the detector's distance from the origin is such that the angle $\alpha \approx \beta$. Keeping the distance fixed at 8000 m, we try with various values of θ.

A.5 (cont.)

		
10°	20°	30°
		
40°	50°	60°
		
70°	80°	90°

We can see that at $\theta = 30^\circ$, far left and right parts of the graph show asymptotic behaviour. In these regions, peak frequencies do not show appreciable change. Notice that the values of the peak frequencies in the left side of the graph is higher than the values of the peak frequencies in the right side of the graph in this region. This indicates that the source is moving away from the detector in the right side of the graph. Detector is placed somewhere in the transient region. Expand the graph for a far left region this gives with a decreased data point interval (say 0.001) for a more accurate $f_{\rm max}$ and $f_{\rm min}$ numbers.
$f_{\rm min} = 788.24$ Hz and $f_{\rm max} = 5569.59$ Hz. Inserting this in Eq. (11)

$$\frac{f_{\rm max} + f_{\rm min}}{f_{\rm max} - f_{\rm min}} = 1.33 = \frac{c - v_s}{\omega R} \tag{18}$$

Far right region gives
$f_{\rm min} = 543.96$ Hz and $f_{\rm max} = 1353.45$ Hz. Inserting this in Eq. (12)

$$\frac{f_{\rm max} + f_{\rm min}}{f_{\rm max} - f_{\rm min}} = 2.34 = \frac{c + v_s}{\omega R} \tag{19}$$

equations (18-19) yields $v_s = 91.1$ m/s and $\omega R = 179.66$ m/s. Also,for any two peaks in asymptotic case

$$\Delta t = 148.84 - 143.48 = 5.36 = \frac{2\pi}{\omega}\left(1 + \frac{v_s}{c}\right) \tag{20}$$

We use the value of $v_s = 91.1$ m/s to get $\omega = 1.49\ \mathrm{rad\ s^{-1}}$. From $\omega R = 179.66$ m/s, $R = 120.57$ m. To obtain f_0, insert $f_{\rm min} = 5327.82$Hz on the far right side in Eq. (8) and solve for f_0. This gives f_0 to be 990.26 Hz.

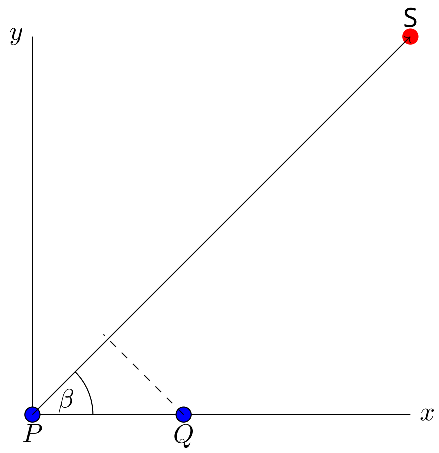
A.5 (cont.)

f_0 (Hz)	ω (s^{-1})	R (m)	v_S (m/s)
990.26Hz	$1.49s^{-1}$	120.57m	91.1m/s

A.6 (2.0 pt)

Calculating β

Figure below represents a schematic picture, where S is a source at a very large distance. P and Q represent two different positions of detectors placed at different instants.



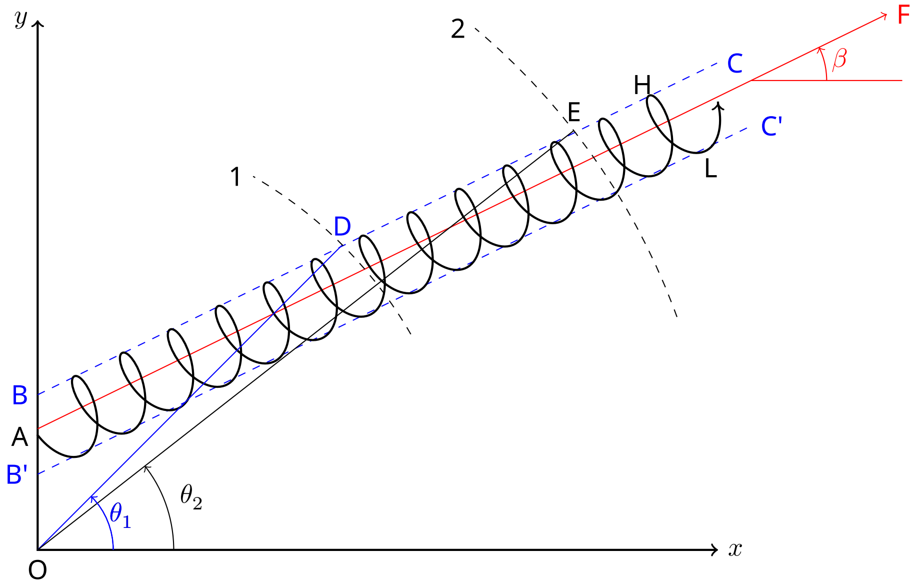
At large distances. let the time taken for the sound signal to reach at P detector: $t_0 = 1009.61$
let the time taken for the sound signal to reach at Q detector:$t_1 = 1007.85$
The distance between P detector and Q detector is 660m and corresponding time taken by sound to reach their respective detectors are 1009.61s and 1007.85s respectively. The expression for time difference is given by

$$t_0 - t_1 = \frac{PQ \cos(\beta)}{c} \tag{21}$$

$$\cos(\beta) = \frac{(t_0 - t_1)c}{PQ} \tag{22}$$

which gives $\beta = 28.36^\circ$

A.6 (cont.)
Alternate solution for β:



Red line AF depicts the direction of the velocity v_s of the circle. We aim to determine β which $\vec{v}_s$ makes with the x-axis.

Value of the frequency detected by the detector depends on two aspects, first from which location on the cycloid, the source emitted the signal and second, on the location of the detector.

Points H and L during one cycle of the source's trajectory depict the location where the source's speed is maximum and minimum respectively. This is due to $\vec{v}_s$ being parallel or anti-parallel to the tangential velocity component of the rotation on these points.

As the source takes n^{th} turn on the cycloid, detector on different angular positions on circular arc 1 will detect different values of $f_{\max}$ corresponding to those positions. Starting from the angular position near the x-axis ($0°$), $f_{\max}$ will keep increasing till the detector is kept on point D at (θ_1). In fact, for any position on line BC which is parallel to AF, the detector will detect maximum of all $f_{\max}$. Similarly, if the detector is placed anywhere on line $B'C'$ which is also parallel to AF, it will detect minimum of $f_{\min}$. In the simulation, you can change the angle by changing x, y coordinates and keeping the velocities zero.

We repeat this exercise by changing the detector distance to arc 2. Scanning across the arc, angle θ_2 can be obtained for which the detector detects maximum of $f_{\max}$.

Once we have the angular positions θ_1 and θ_2 determined, we can use the coordinates of point D and E to calculate the angle of segment DE which it makes with the x-axis. This is the angle β. If the coordinates of point D and E are (x_1, y_1) and (x_2, y_2) respectively. Then

$$\beta = \text{arctan}\,\frac{y_2 - y_1}{x_2 - x_1} \tag{23}$$

A.6 (cont.)

This process is illustrated in table below and the corresponding graph. First we place the detector at 8000 m away from the origin and change the coordinates for the corresponding angular position 0° - 90°. We record $f_{\max}$ for any fixed cycle, (10th in this case). It can be seen from the plot of $f_{\max}$ vs θ that the θ_1 is between 25° - 35°.

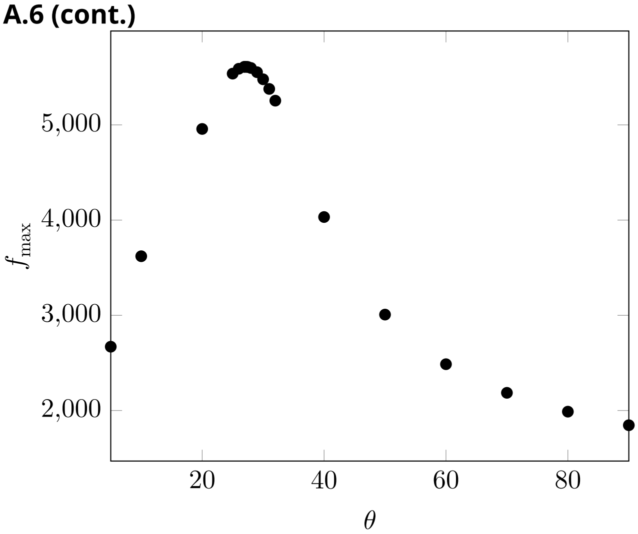
θ	$f_{\min}$	$f_{\max}$
5	676.08	2670.30
10	722.99	3620.51
20	763.49	4957.28
30	781.46	5478.86
40	753.98	4032.21
50	711.98	3007.44
60	677.39	2486.46
70	651.25	2185.81
80	630.99	1987.99
90	614.68	1845.65

A.6 (cont.)
We go in smaller steps to determine θ_1 more accurately. Figure below shows the table and graph for the variation between $25^\circ - 35^\circ$.

θ	$f_{\min}$	$f_{\max}$
25	777.95	5538.23
26	779.67	5589.40
26.9	780.80	5609.15
27	780.90	5609.74
27.3	781.18	5609.546
27.5	781.33	5607.78
28	781.62	5597.66
29	781.81	5553.37
30	781.46	5478.86
31	780.58	5377.65
32	779.17	5254.35

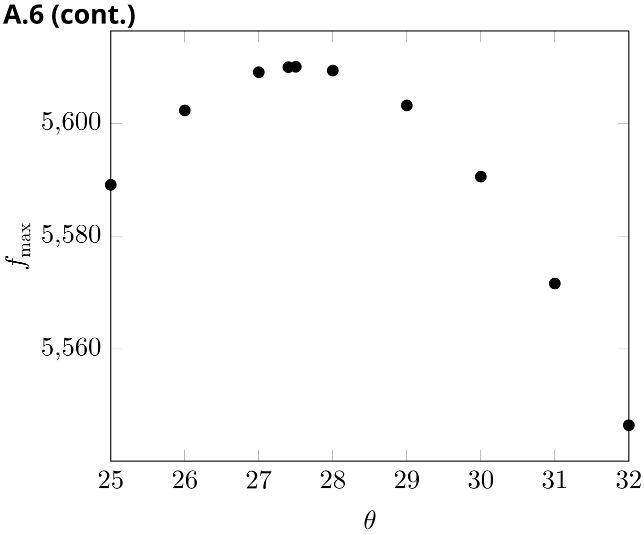
It is clear from the table and graph that $\theta_1 = 27^\circ$. We repeat this for another distance 16000 m. Table and graph for this distance is given below.

A.6 (cont.)



θ	$f_{\min}$	$f_{\max}$
25	781.12	5589.093
26	781.47	5602.27
27	781.70	5609.05
27.4	781.76	5609.95
27.5	781.77	5610.01
28	781.81	5609.34
29	781.80	5603.16
30	781.66	5590.544
31	781.41	5571.60
32	781.04	5546.48

A.6 (cont.)



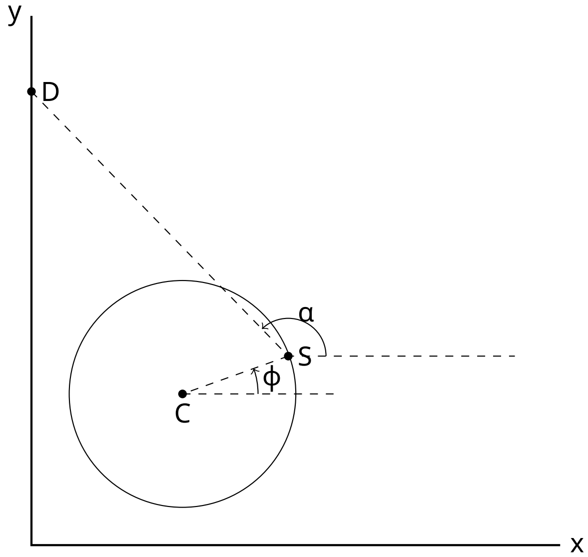
This gives $\theta_2 = 27.5^\circ$. Taking the corresponding values of x_1, y_1, x_2, y_2 and using Eq. (23),

$$\beta = \text{arctan}\,\frac{7468.77 - 3631.92}{14192.17 - 7128.052194} = 28.5^\circ \tag{24}$$

A.7 (2.1 pt)

Coordinates of the center of the circle

For this part, keep the detector at some fixed position say on the y-axis. A schematic diagram of the initial location of the source and the the detector is depicted in the figure below (figure is not to scale).



We record the detected frequency of the first signal sent by the source. We already have the value of R and the source's initial coordinates. The detected frequency at $t = 0$, i.e. the first signal is 795.69 Hz if the detector is kept at 500 m on the y-axis. With the source's coordinates (419.99,499.99) m,

$$\tan(180 - \alpha) = \frac{0}{419.99} \tag{25}$$

$$\alpha = 180^\circ \tag{26}$$

Using the values of detected frequency and α in Eq. (7)

$$795.69 = \frac{990.26 \times 330}{330 - 91.1\cos(28.5 - 180) + 179.66\sin(\phi - \alpha)} \tag{27}$$

$$\Rightarrow \phi \approx 0^\circ \tag{28}$$

This yields source's center coordinates to be (299.42,499.99) m.

EQ2: Acoustic black box marking Scheme			
Section		**Partwise marks**	**Total**
A1	**finding expression of** $x_(t)$ **and** $y_(t)$		**0.2**
A2	**Plotting graph of** $f_{\min}$ **vs** t		**1.2**
	Choice of scale (70% coverage)	0.2	
	Both axis labelled with proper units	0.2	
	More than 8 points labelled correctly	either 0.4	
	Atleast 5-7 points plotted	or 0.2	
	Less than 5 points plotted	**No Credit**	
	Data table		
	10 points reported correctly	either 0.4	
	6-9 points reported correctly	or 0.2	
	Less than 6 points reported	**No Credit**	
A3	**Evaluating expression of eventual** $f_{\min}$(No partial marking)		**1**
A4	**Determination of source's coordinates**		**1.4**
	idea of triangulation (i.e. getting the equations correctly)	**1**	
	correct value of Δt_{x1} and Δt_{x2}	0.1+0.1	
	Final calculation of X_A and Y_A	0.1+0.1	
A5	**Calculating** f_0, ω, R, v_s		**2.1**
	Logic asymptotic values	0.3	
	Getting the retarded time expression correctly	0.6	
	Value of ω	0.2	
	expression of $f_{\max}$ and $f_{\min}$ for source moving away	0.3	
	expression of $f_{\max}$ and $f_{\min}$ for source moving towards	0.3	
	Determination of correct $f_{\max}$ and $f_{\min}$ for calculation	0.2	
	If all values correct f_0, R,v_s	either 0.2	
	If only two or one value is correct	or 0.1	

ABB Marking Scheme			
A6	**Finding angle β**		**2**
	Calculation of largest maximum frequency at some angle	0.6	
	Reporting data of extrema at various θ	0.4	
	Determination of the coordinates of D	0.4	
	Determination of Coordinates of E	0.4	
	Final calculation of β	0.2	
A7	**Finding centre coordinates**		**2.1**
	Expression of $f(t')$	1	
	If t' and t are same	-0.5	
	Determination of α	0.4	
	Determination of ϕ	0.4	
	coordinates of centre of circle	0.3	