Первый Контест Symmetrix: Старшая Лига

1. Prove that for every positive integer n there exists an n-digit number divisible by 5^n all of whose digits are odd.

2. A convex polygon $\mathcal{P}$ in the plane is dissected into smaller convex polygons by drawing all of its diagonals. The lengths of all sides and all diagonals of the polygon $\mathcal{P}$ are rational numbers. Prove that the lengths of all sides of all polygons in the dissection are also rational numbers.

3. Let $n \neq 0$. For every sequence of integers

$$A = a_0, a_1, a_2, \dots, a_n$$

satisfying $0 \leq a_i \leq i$, for $i = 0, \dots, n$, define another sequence

$$t(A) = t(a_0), t(a_1), t(a_2), \dots, t(a_n)$$

by setting $t(a_i)$ to be the number of terms in the sequence A that precede the term a_i and are different from a_i. Show that, starting from any sequence A as above, fewer than n applications of the transformation t lead to a sequence B such that $t(B) = B$.