

## The 2nd International Olympiad on Astronomy and Astrophysics Bandung, Indonesia

## Saturday, 23 August 2008 Theoretical Competition

## Please read this carefully:

1. Every student receives problem sheets in English and/or in native language, an answer book and a scratch book.
2. The time available is 5 hours for the theoretical competition. There are fifteen short questions (Theoretical Part 1), and three long questions (Theoretical Part 2).
3. Use only Black or dark blue pen
4. Use only the front side of answer sheets. Write only inside the boxed area.
5. Begin answering each question on a separate sheet.
6. Numerical results should be written with as many digits as are appropriate.
7. Write on the blank answer sheets whatever you consider is required for the solution of each question. Please express your answer primarily in term of equations, numbers, figures, and plots. If necessary provide your answers with concise text. Full credit will be given to correct answer with detailed steps for each question. Underline your final result.
8. Fill in the boxes at the top of each sheet of paper with your country code and your student code.
9. At the end of the exam place the books inside the envelope and leave everything on your desk.

## Astronomical and Physical Constants

| Quantity | Value |
| :---: | :---: |
| Astronomical unit (AU) | 149597870691 m |
| Light year (ly) | $9.4605 \times 10^{15} \mathrm{~m}=63,240 \mathrm{AU}$ |
| Parsec (pc) | $3.0860 \times 10^{16} \mathrm{~m}=206,265 \mathrm{AU}$ |
| Sidereal year | 365.2564 days |
| Tropical year | 365.2422 days |
| Gregorian year | 365.2425 days |
| Sidereal month | 27.3217 days |
| Synodic month | 29.5306 days |
| Mean sidereal day | $23^{\mathrm{h}} 56^{\mathrm{m}} 4^{\mathrm{s}} .091$ of mean solar time |
| Mean solar day | $24^{\mathrm{h}} 3^{\mathrm{m}} 56^{\mathrm{s}} .555$ of sidereal time |
| Mean distance, Earth to Moon | 384399000 m |
| Earth mass ( $\mathrm{M}_{\oplus}$ ) | $5.9736 \times 10^{24} \mathrm{~kg}$ |
| Earth mean radius | 6371000 m |
| Earth mean velocity in orbit | $29783 \mathrm{~m} / \mathrm{s}$ |
| Moon mass ( $\mathrm{M}_{\nu}$ ) | $7.3490 \times 10^{22} \mathrm{~kg}$ |
| Moon mean radius | 1738000 m |
| Sun mass ( $\mathrm{M}_{\odot}$ ) | $1.9891 \times 10^{30} \mathrm{~kg}$ |
| Sun radius ( $R_{\odot}$ ) | $6.96 \times 10^{8} \mathrm{~m}$ |
| Sun luminosity ( $L_{\odot}$ ) | $3.96 \times 10^{26} \mathrm{~J} \mathrm{~s} \mathrm{~s}^{-1}$ |
| Sun effective temperature ( $T_{\text {eff }}$ ) | $5800{ }^{\circ} \mathrm{K}$ |
| Sun apparent magnitude ( $m_{\odot}$ ) | -26.8 |
| Sun absolute magnitude ( $M_{\odot}$ ) | 4.82 |
| Sun absolute bolometric magnitude ( $\mathrm{Mbol}_{\text {¢ }}$ ) | 4.72 |
| Speed of light (c) | $2.9979 \times 10^{8} \mathrm{~m} / \mathrm{s}$ |
| Gravitational constant ( $G$ ) | $6.6726 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} \mathrm{~kg}^{-2}$ |
| Boltzmann constant (k) | $1.3807 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1}$ |
| Stefan-Boltzmann constant ( $\sigma$ ) | $5.6705 \times 10^{-8} \mathrm{~J} \mathrm{~s}^{-1} \mathrm{~m}^{-2} \mathrm{~K}^{-4}$ |
| Planck constant ( $h$ ) | $6.6261 \times 10^{-34} \mathrm{~J} \mathrm{~s}$ |
| Electron charge (e) | $1.602 \times 10^{-19} \mathrm{C}=4.803 \times 10^{-10} \mathrm{esu}$ |
| Electron mass ( $m_{e}$ ) | $5.48579903 \times 10^{-4} \mathrm{amu}=9.11 \times 10^{-31} \mathrm{~kg}$ |
| Proton mass ( $m_{p}$ ) | $1.00727647 \mathrm{amu}=1.67268 \times 10^{-27} \mathrm{~kg}$ |


| Neutron mass $\left(m_{n}\right)$ | $1.008664904 \mathrm{amu}=1.67499 \times 10^{-27} \mathrm{~kg}$ |
| :--- | :--- |
| Deuterium nucleus mass $\left(m_{d}\right)$ | $2.013553214 \mathrm{amu}=3.34371 \times 10^{-27} \mathrm{~kg}$ |
| Hydrogen mass | $1.00794 \mathrm{amu}=1.67379 \times 10^{-27} \mathrm{~kg}$ |
| Helium mass | $4.002603 \mathrm{amu}=1.646723 \times 10^{-27} \mathrm{~kg}$ |
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| Conversion table |  |
| :--- | :--- |
| $1 \AA$ | $0.1 \mathrm{~nm}=10^{-10} \mathrm{~m}$ |
| $1 \AA$ | $10^{-28} \mathrm{~m}^{2}$ |
| 1 barn | $10^{-4} \mathrm{~T}$ |
| 1 G | $10^{-7} \mathrm{~J}=1$ dyne cm |
| 1 erg | $3.3356 \times 10^{-10} \mathrm{C}$ |
| 1 esu | $1.6606 \times 10^{-27} \mathrm{~kg}$ |
| 1 amu (atomic mass unit) | $101,325 \mathrm{~Pa}=1.01325$ bar |
| 1 atm (atmosphere) | $10^{-5} \mathrm{~N}$ |
| 1 dyne |  |

## THEORETICAL PART 2

(300 points for 3 Theoretical Part-2, 100 points for each question)
Show your method of solution step by step in the answer sheets completely as your final answer. The scratch sheet is to be used for your personal calculation and will not be marked. Partial credits will be given for answers without showing method of solution.
1.

An eclipsing binary star system has a period of 30 days. The light curve in the figure below shows that the secondary star eclipses the primary star (from point A to point D ) in 8 hours (measured from the time of first contact to final contact), whereas from point B to point C , the total eclipse period is 1 hour and 18 minutes. The spectral analysis yields the maximum radial velocity of the primary star to be $30 \mathrm{~km} / \mathrm{s}$ and of the secondary star to be $40 \mathrm{~km} / \mathrm{s}$. If we assume that the orbits are circular and has an inclination of $i=90^{\circ}$, determine the radii and the masses of both stars in unit of solar radius and solar mass.

2. A $U B V$ photometric ( $U B V$ Johnson's) observation of a star gives $U=8.15, B=8.50$, and $V=8.14$. Based on the spectral class, one gets the intrinsic color $(U-B)_{o}=-0.45$. If the star is known to have radius of $2.3 R_{\odot}$, absolute bolometric magnitude of -0.25 , and bolometric correction $(B C)$ of -0.15 , determine:
a. the intrinsic magnitudes $U, B$, and $V$ of the star (take, for the typical interstellar matters, the ratio of total to selective extinction (color excess) $R_{V}=3.2$ ),
b. the effective temperature of the star,
c. the distance to the star in pc.

Note: The relation between color excess of $U-B$ and of $B-V$ is $E(U-B)=0.72 E(B-V)$.

Let $A_{v}$ be the interstellar extinction and $R=3.2$, then $A_{v}=3.2 E(B-V)$.
3. Measurement of the cosmic microwave background radiation (CMB) shows that its temperature is practically the same at every point in the sky to a very high degree of accuracy. Let us assume that light emitted at the moment of recombination ( $T_{r} \approx$ $3000 \mathrm{~K}, t_{r} \approx 300000$ years) is only reaching us now ( $T_{\mathrm{o}} \approx 3 \mathrm{~K}, t_{\mathrm{o}} \approx 1.5 \times 10^{10}$ years). Scale factor $S$ is defined as such $S_{0}=S\left(t=t_{o}\right)=1$ and $S_{t}=S\left(t<t_{o}\right)<1$. Note that the radiation dominated period was between the time when the inflation stopped ( $t=10^{-32}$ seconds) and the time when the recombination took place, while the matter dominated period started at the recombination time. During the radiation dominated period $S$ is proportional to $t^{1 / 2}$, while during the matter dominated period $S$ is proportional to $t^{2 / 3}$.
a. Estimate the horizon distances when recombination took place. Assume that temperature $T$ is proportional to $1 / S$, where $S$ is a scale factor of the size of the Universe.
b. Note: Horizon distance in degrees is defined as maximum separation between the two points in CMBR imprint such that the points could "see" each other at the time when the CMBR was emitted.
c. Consider two points in CMBR imprint which are currently observed at a separation angle $\alpha=5^{\circ}$. Could the two points communicate with each other using photon? (Answer with "YES" or "NO" and give the reason mathematically)
d. Estimate the size of our Universe at the end of inflation period.

