THEORETICAL PART 2

1. The coordinates of the components of Visual Binary Star μ Sco on August 22, 2008 are given in the table below

	α (RA)	δ (Dec)
μ Sco 1 (primary)	$20^h\ 17^m\ 38^s.90$	-12° 30' 30"
μ Sco 2 (secondary)	$20^h\ 18^m\ 03^s.30$	-12° 32' 41"

The stars are observed using Zeiss refractor telescope at the Bosscha Observatory with aperture and focal length are 600 mm and 10,780 mm, respectively. The telescope is equipped with 765 × 510 pixels CCD camera. The pixel size of the chip is 9 μm × 9 μm.

a. Can both components of the binary be inside the frame? ("YES" or "NO", show it in your computation!)

b. What is the position angle of the secondary star, with respect to the North?

Solution:

a. Chip size: (9×765) μm ×(9×510) μm = 6885 μm × 4590 μm

≈ 6.9 mm × 4.6 mm (10%)

For Zeiss telescope: FL = 10780 mm

Pixel size (arc sec) = 206265 × pixel size (μm)/(1000 × focal length (mm))" (10%)

= 206265 × 9/(1000 × 10780) = 0.1722" (10%)

FOV CCD chip = (765 × 510) × 0.1722 = 131.7330 × 87.8220"

= 2.1956 × 1.4637' (10%)

For μ Sco 1 : $\alpha_1 = 20^h\ 17^m\ 38^s.90 = 20.2941^h = 304.4121^\circ$

$\delta_1 = -12^\circ\ 30'\ 30'' = -12.5083^\circ$

For μ Sco 2 : $\alpha_2 = 20^h\ 18^m\ 03^s.30 = 20.3009^h = 304.5138^\circ$

$\delta_2 = -12^\circ\ 32'\ 41'' = -12.5447^\circ$

Let the angular separation between the stars be γ

Use spherical trigonometry :

$$\cos\gamma = \cos(90^\circ - \delta_1)\cos(90^\circ - \delta_2) + \sin(90^\circ - \delta_1)\sin(90^\circ - \delta_2)\cos(\alpha_2 - \alpha_1) \quad (10\%)$$

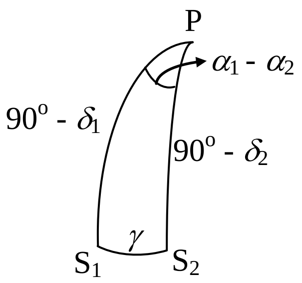
$$\cos\gamma = \cos(90^\circ - (-12^\circ.5083))\cos(90^\circ - (-12^\circ.5447)) +$$

$$\sin(90^\circ - (-12^\circ.5083))\sin(90^\circ - (-12^\circ.5447))\cos(304.5138^\circ - 304.4121^\circ)$$

$$\cos\gamma = \cos(102.5083^\circ)\cos(102.5447^\circ) +$$

$$\sin(102.5083^\circ)\sin(102.5447^\circ)\cos(0.1017^\circ) = 0.999998298 \quad (10\%)$$

$$\gamma = 0.1057^o = 6.3424'$$



NO, because $\gamma > \sqrt{2.1956^2 + 1.4637^2}$ (diagonal of FOV Chip). (10%)

b. Position angle (S_1):

$$\frac{\sin(\alpha_2 - \alpha_1)}{\sin\gamma} = \frac{\sin S_1}{\sin(90^0 - \delta_2)}$$

$$\sin S_1 = \frac{\sin(0^o 6' 6'')\sin(102^o 32' 41'')}{\sin(6'.1020)} = 0.9388$$

S_1 = 69.8535° or 110.1466° (20%)

From the figure above, we found that the correct position angle is 110.15° (10%)

(± 6° is full mark)

2. A *UBV* photometric (*UBV* Johnson's) observation of a star gives U = 8.15, B = 8.50, and V = 8.14. Based on the spectral class, one gets the intrinsic color $(U - B)_o$ = -0.45. If the star is known to have radius of 2.3 $R_\odot$, absolute bolometric magnitude of -0.25, and bolometric correction (BC) of -0.15, determine:

a. the intrinsic magnitudes U, B, and V of the star (take, for the typical interstellar matters, the ratio of total to selective extinction R = 3.2),

b. the effective temperature of the star,

c. the distance to the star.

Solution:

U = 8.15, B = 8.50, and V = 8.14.

$(U - B)_o$ = - 0.45. R = 2.3 $R_\odot$ = 1.60 x 10^{11} cm ,

M_{bol} = - 0.25 , BC = - 0.15

a) $U - B$ = 8.15 – 8.50 = - 0.35

The color excess for U-B:

$E(U - B) = (U - B) - (U - B)_o$ = - 0.35 – (-0.45) = 0.10

The relation between color excess of U - B and of B - V:

$E(U - B) = 0.72\ E(B - V) \Longrightarrow E(B - V) = 0.10/0.72 = 0.14$ (15%)

Let A_v be the interstellar extinction and R = 3.2, then

$A_v = 3.2\ E(B\text{-}V) = 3.2\ (0.14) = 0.45$

$V - V_o = A_v \Longrightarrow V_o = V - A_v = 8.14 - 0.45 = 7.69$

$E(B - V) = (B - V) - (B - V)_o \Longrightarrow (B - V)_o = (B - V) - E(B - V)$

$= (8.50 - 8.14) - 0.14 = 0.22$ (15%)

$(B - V)_o = B_o - V_o = 0{,}22 \Longrightarrow B_o = 0.22 + V_o = 0.22 + 7.69 = 7.91$ (10%)

$(U - B)_o = U_o - B_o = -\ 0{,}45 \Longrightarrow U_o = B_o - 0.45 = 7.91 - 0.45 = 7.46$ (10%)

b) Luminosity - absolute magnitude relation:

$M_{bol} - M_{bol\odot} = -2.5 \log L/L_\odot \Longrightarrow L/L_\odot = 10^{-(M_{bol} - M_{bol\odot})/2.5}$

$= 10^{-(-0{,}25-4{,}75)/2{,}5}$

$= 100$ (10%)

$L = 100\ L_\odot = 3.90 \times 10^{35}$ erg/det

$L = 4\pi\sigma R^2 T_{ef}^4$

$$T_{ef} = \left(\frac{L}{4\pi\sigma R^2}\right)^{1/4} = \left(\frac{3.90\times10^{35}}{4\pi(5.67\times10^{-5})(1.60\times10^{11})^2}\right)^{1/4} = 12\ 092\ \text{K}$$ (15%)

c) $M_v - M_{bol} = BC \Longrightarrow M_v = M_{bol} + BC = -0.25 - 0.15 = -0.40$ (10%)

$m_v - M_v = -5 + 5 \log d + A_v \Longrightarrow 5 \log d = m_v - M_v + 5 - A_v$

$= 8.14 + 0.40 + 5 - 0.45 = 13.09$

$d = 10^{13.09/5} = 10^{2.62} = 414.9$ pc (15%)

3. Measurement of the cosmic microwave background radiation (CMB) shows that its temperature is practically the same at every point in the sky to a very high degree of accuracy. Let us assume that light emitted at the moment of recombination ($T_r \approx$ 3000 K, $t_r \approx$ 300000 years) is only reaching us now ($T_o \approx$ 3 K, $t_o \approx 1.5 \times 10^{10}$ years). Scale factor S is defined as such $S_0 = S(t = t_o) = 1$ and $S_t = S(t < t_o) < 1$. Note that the radiation dominated period was between the time when the inflation stopped

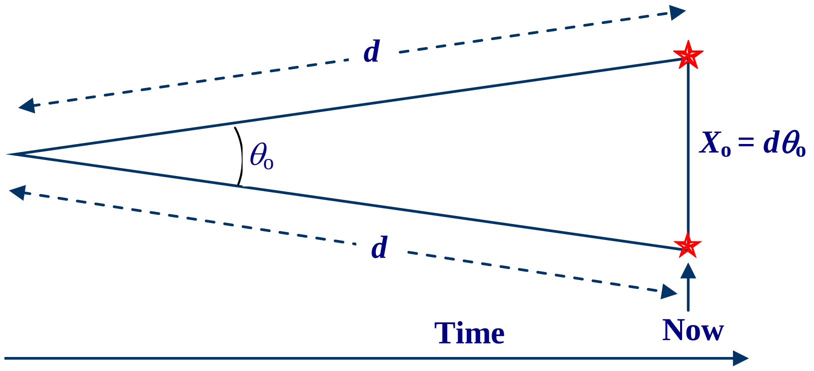
(t = 10^{-32} seconds) and the time when the recombination took place, while the matter dominated period started at the recombination time. During the radiation dominated period S is proportional to $t^{1/2}$, while during the matter dominated period S is proportional to $t^{3/2}$.

a. Estimate the horizon distances when recombination took place. Assume that temperature T is proportional to $1/S$, where S is a scale factor of the size of the Universe.

b. Consider two objects which are currently observed at a separation angle α = 5°. Could the two objects communicate each other using photon? (Answer with "YES" or "NO" and give the reason mathematically)

c. Estimate the size of our Universe at the end of inflation period.

Solution:



a) $X_0 = d\theta_0$

where $d = c(t_o - t_r) \approx ct_{o,}\ t_o >> t_r$.

Scale factor S is defined as such $S_0 = S(t = t_o) = 1$ and $S_t = S(t < t_o) < 1$.

Maximum distance between objects at the time of recombination

$$X_r = S_r X_o = S_r \theta_o t_o c$$

On the other hand $X_r = ct_r$, such that

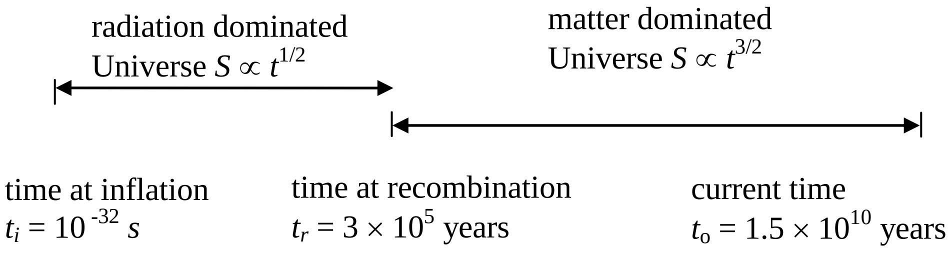
$$ct_r = S_r \theta_0 t_o c \qquad \text{or} \qquad \theta_o = \frac{t_r}{S_r t_o} \qquad (20\%)$$

From $T \propto \frac{1}{S}$ then $\frac{S_o}{S_r} = \frac{T_r}{T_o}$ and $\frac{1}{S_r} = \frac{T_r}{T_o}$.

$$\text{So,} \quad \theta_o = \frac{t_r T_r}{t_o T_o} = \frac{(3\times10^5\,yr)(3000\,K)}{(1.5\times10^{10}\,yr)(3\,K)} = 2\times10^{-2}\,rad = 1.14^\circ \qquad (20\%)$$

b) NO, because $\alpha > \theta_o$ (20%)

c)



From the time at recombination (t_r) to the current time (t_0) $S \propto t^{2/3}$, we derive

$$\frac{S_o}{S_r} = \left(\frac{t_o}{t_r}\right) = \left(\frac{1.5\times10^{10}\, years}{3\times10^{5}\, years}\right)^{2/3} = 1.36\times10^{3}$$

The size of our universe at t_r is

$$\frac{1.5\times10^{10}\, light\ years}{1.36\times10^{3}} = 1.11\times10^{7}\, light\ years$$ (20%)

When the inflation stopped, the Universe was radiation dominated $\left(t_r = 3\times10^{5}\, years \text{ to } t_i = 10^{-32}\, s\right)$, we get

$$\frac{S_r}{S_i} = \left(\frac{t_r}{t_i}\right)^{1/2} = \left(\frac{9.46\times10^{12}\, s}{10^{-32}\, s}\right)^{1/2} = 3.08\times10^{22}.$$

The size of universe at the end of the inflation period was

$$\frac{1.1\times10^{7}\, light\ years}{3.08\times10^{22}} = 3.57\times10^{-16}\, light\ years \approx 3\, m$$ (20%)