Solution

1) Comparison of Figs. 7.1 and 7.2 allow us to identify the asteroid and evaluate its angular displacement due to its motion. We put the transparency on Fig.7.1 and mark positions of several bright objects with a pencil. Then we put the transparency on Fig. 7.2 and move it until the markings of objects on the transparency will coincide with positions of corresponding objects in Fig. 7.2. We notice that one object changed its position considerably in Fig. 7.2. This is the asteroid.
2) We mark the position of the asteroid on the transparency. Then we measure the displacement of the asteroid on the transparency and get Δl =44 mm. Using the scale of Fig. 7.1 we calculate the angular displacement of the asteroid.

$\Delta\theta=180\times44/81=98$ arcsec

The time interval between images of Figs.7.1 and 7.2 is Δt =7h16m-4h53m=2h23m=8580 s.

The angular velocity of the asteroid $\mu=\Delta\theta/\Delta t=98/8580=0.011$ arcsec/s.

3) The parallax of the asteroid is evaluated by comparison of Figs. 7.3 and 7.4. These two images were taken from two different locations at the same time. We mark the positions of the asteroid overlaying the same transparency on Figs. 7.3 and 7.4. and measure the displacement of the asteroid on the same transparency, $\Delta b=7$ mm.

   Angular displacement of the asteroid is $\Delta\phi=180\times7/81=16$ arcsec. Then the parallax $p=\Delta\phi/2=8$ arcsec.

The distance of the asteroid is $d=B/(2\tan p)$, where B is the baseline, i.e. the distance between observatories. Then $d=206265\times B/(2\,p)$ since p is a small angle and measured in arcsec.

$d=206265\times3172/(2\times8)$ = 40900000 km = 0.27 au.

4) The tangential linear velocity of the asteroid is $v = d\tan\mu$ or $v=d\mu/206265$.

$v=40900000\times0.011/206265$ = 2 km/s.

Solution

Table A

N [nm/mm] =	(659.39-654.62)/215.5 ≈ **0.0221**

λ [nm]	dx [mm]	$d\lambda$[nm]	v_r [km/s]
654.62	~5	~0.1105	~12.65
656.92	~5	~0.1105	~12.61
659.26	~5	~0.1105	~12.56
		v_r_avg [km/s] =	**~12.6**

FORMULAE

Because of the Doppler effect the wavelength shift , $d\lambda$, observed at the edge of Jupiter's disk can be obtained through the following formula:

$$\mathbf{v_r = \frac{1}{4} \cdot \frac{d\lambda}{\lambda} \cdot c}$$

,where c – speed of light in the vaccum.

Coefficient $\frac{1}{4}$ comes into effect due to the fact that:

-the sunlight, illuminating the planet, is reflected, therefore it was affected by Doppler effect twice,

- measurements are done at the edges of the disk and not relative to the non-shifted center, thus doubling the measured shift value.

Table B

dt[s] =	6695

Feature	x_1 [mm]	x_2 [mm]	L_x [mm]	ϕ [°]
1	66	33	92	66.86
*2**	*57*	*46*	*92.5*	*67.86*
*3**	*44*	*54*	*87*	*68.75*
			ϕ_avg =	**~ 67.8**

**only as example - acctual values depends on choosen feature and accuracy of measuremets*

P_{Je} [h] =	**~ 9.9**
R_{Je} [km] =	**~71470**

FORMULAE

$$\varphi = \arcsin\left(\frac{x_1}{L_x}\right) + \arcsin\left(\frac{x_2}{L_x}\right)$$

$$P = \frac{360°}{3600\ s} \times \frac{dt}{\varphi}$$

$$R_{Je} = \frac{v_r \cdot P_{Je}}{2\pi}$$

Table C

Moon	P_m [h]	a_Je	a [m]	M_J [kg]
1	~ 43	~ 5.9	$\sim 4.22 \times 10^8$	$\sim 1.86 \times 10^{27}$
2	~ 85	~ 9.4	$\sim 6.70 \times 10^8$	$\sim 1.90 \times 10^{27}$
3	~ 172	~ 14.9	$\sim 10.65 \times 10^8$	$\sim 1.86 \times 10^{27}$
			M_J_avg [kg] =	**$\sim 1.87 \times 10^{27}$**

R_p/R_e =	174 mm / 186 mm ≈ **0.9355**
R_J_avg [m] =	**$\sim 69900 \times 10^3$**
V [m^3] =	**$\sim 1.43 \times 10^{24}$**
ρ_J [kg/m^3] =	**~ 1310**

FORMULAE

In case of a moon orbiting a much more massive planet, the mass of the central body (planet) can be obtained from Kepler`s Third Law:

$$\mathbf{M_J} = \frac{\mathbf{4\pi^2}}{\mathbf{G}} \cdot \frac{\mathbf{a^3}}{\mathbf{P^2}}$$

, where G – gravitational constant, a – semi-major axis of moon orbit in meters, P – period in seconds

The mean radius for a slightly oblate ellipsoid is:

$$\mathbf{R_J\_avr} = \sqrt[3]{\frac{R_p}{R_e} R_e R_e^2} = \mathbf{R_e} \times \sqrt[3]{\mathbf{R_p/R_e}}$$

Distance to the galaxy NGC 4214

Solution

7.1. Distances of the novae from Table 1 are calculated using their shell radii assuming that their rates of expansion are constant. Linear radius of the nova's shell:

$$R = v \times \Delta t \,,$$

where v is the expansion rate of the nova's shell , and Δt is the time interval that elapsed from the nova's outburst up to time when the angular radius of nova's shell have been measured. Radii of the shells should be expressed in astronomical units (au). Since 1au=1.496 $\cdot 10^8$ km and 1day=24h $\cdot$3600s=86400s we get

$$R(\mathrm{au}) = \frac{86400}{1.496 \cdot 10^8} v\Delta t = 5.775 \cdot 10^{-4} v\Delta t$$

Distance to a nova is calculated by the formula:

$$d = \frac{R}{\tan\theta}$$

Since θ is a small angle and is measured in arcsec, and R is expressed in au, we get the distance expressed in parsecs:

$$d = \frac{R}{\theta}$$

Then absolute magnitudes of the novae are calculated by the formula:

$$M_V = m_V - 5\log d + 5 - A_V$$

Table 1a. Results of calculations of the parameters of the galactic novae

No.	Δt (JD)	R (au)	d (pc)	$M_{V\max}$	$\log t_2$
1	32715	11336	1260	-7.3	1.65
2	3052	2644	1763	-10.7	0.30
3	17004	4910	491	-7.2	1.59
4	7646	4857	1388	-7.8	1.34
5	16458	15207	1382	-9.5	0.70
6	20222	9343	1038	-9.7	0.78

Now we plot the graph $\log t_2; M_{V\max}$ and draw the straight line best fitted through the points (Fig. 1b).

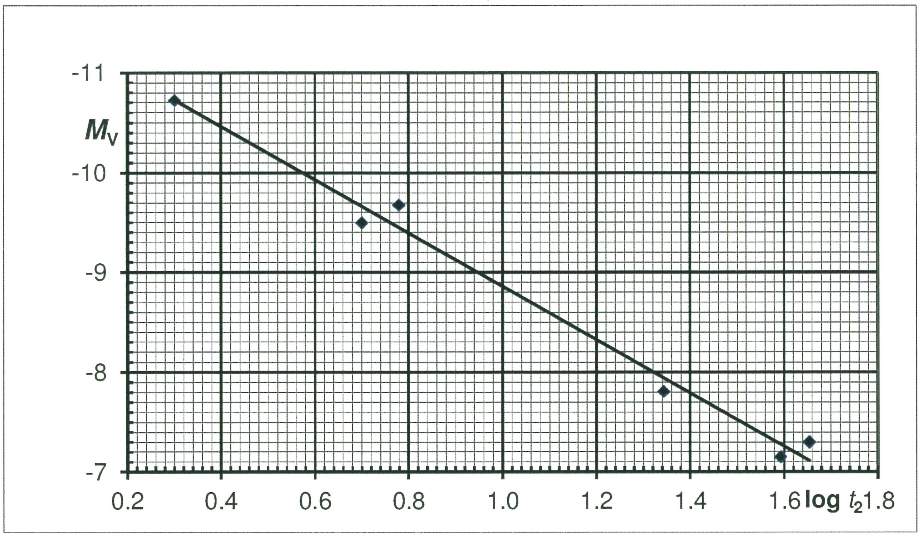

Fig. 1b. Graph for determination of the constants in the expression (1)

The equation of the line is

$$M_{V\max} = -11.53 + 2.67 \log t_2 \tag{1a}$$

7.2. We use the data of Table 2 to plot the light curve of the nova in NGC4214 (Fig. 2).

Fig. 2b. Light curve of the nova NGC 4214

From this graph we see that the nova reached its light maximum, m_V=17.3, at JD=2455233.8. Brightness of the nova dropped by 2 mag from its maximum at about JD=2455237.35. So we get the time $t_2 = 2455237.35 - 2455233.8 = 3.55$ days.

Using the relation (1a) we get the absolute magnitude of the nova NGC 4214 at its maximum

$$M_{V\max} = -11.53 + 2.67 \log 3.55 = -10.06$$

We assume that interstellar extinction in the direction of the nova (as well as NGC 4214) is negligible and calculate distance to the galaxy NGC 4214:

$$\log d = \frac{m_{V\max} - M_{V\max} + 5}{5}$$

$$\log d = \frac{17.3 + 10.06 + 5}{5} = 6.47$$

$$d \approx 3.0 \times 10^6 \text{ pc}$$

***<u>Answer:</u>* Distance to the galaxy NGC 4214 is 3.0 Mpc.**